



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2019

First Degree Programme under CBCSS

MATHEMATICS

MM 1661.3 : Complex Integration (Elective)

(2013 Admn.)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are **compulsory**. They carries 1 mark each.

1. Evaluate $\int_0^{\infty} e^{-zt} dt$ ($\operatorname{Re} z > 0$).
2. Evaluate $\int_C \frac{e^z}{z+2i} dz$ where C is the circle $|z-i|=2$.
3. Let C be the circle $|z|=3$ described in the positive sense. Show that if $g(z) = \int_C \frac{2s^2 - s - 2}{s-z} ds$, then $g(2) = 8\pi i$.
4. Find the singular point of the function $\frac{1}{z}$ and specify the type of singularity.
5. What type of singularity that $f(z) = \exp\left(\frac{1}{z}\right)$ have at the origin ?
6. The Laurent's series expansion of $f(z) = \exp\left(\frac{1}{z^2}\right)$ is $1 + \frac{1}{z^2} + \frac{1}{2!z^4} + \frac{1}{3!z^6} + \dots$ about the point $z=0$. Find the residue of $f(z)$ at the isolated singular point $z=0$.
7. Why $z=0$ is not an isolated singular point of $\operatorname{Log} z$?
8. Find the residue at the simple pole $z=n\pi$ of the function $f(z) = \cot z$.
9. Find the singularities of the function $f(z) = \frac{\sin z}{z}$ and classify the singularity.
10. Write the Cauchy principle value (PV) of $\int_{-\infty}^{\infty} f(x) dx$, if $f(x)$ is even.

P.T.O.



SECTION - II

Answer **any 8** questions from among the questions **11 to 22**. These questions carry **2 marks each** :

11. Use an antiderivative $F(z)$ of the integrand to evaluate $\int_0^{1+i} z^2 dz$.
12. Find the length of the unit circle given by $z(t) = \cos t + i \sin t$ ($0 \leq t \leq 2\pi$).
13. Evaluate $\int_C \bar{z} dz$ where C is the upper half of the unit circle $|z| = 1$ from $z = -1$ to $z = 1$.
14. Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$ where C is described in the positive sense.
Evaluate $\int_C \frac{z dz}{2z+1}$.
15. Evaluate $\int_C \frac{e^{iz}}{z^3} dz$ where C is the circle $|z| = 1$.
16. If a function f is continuous throughout a simply connected domain, and if, for each simple closed contour C lying in D , $\int_C f(z) dz = 0$, then f is analytic throughout D .
17. Determine and classify the singular points of $f(z) = \frac{z}{e^z - 1}$.
18. Using Cauchy's integral formula, evaluate $\frac{1}{2\pi i} \int_C \frac{z^2 + 5}{z - 3} dz$ where C is $|z| = 4$.
19. Write the principal part of the function $f(z) = \sin\left(\frac{1}{z}\right)$ at its isolated singular point and determine the type of singularity.
20. Find the residue of $f(z) = \frac{z+1}{z^2 - z}$ at its pole $z = 0$.



21. Use Residue Theorem to evaluate $\int_C \frac{dz}{z^2 e^z}$ where C is the circle $|z| = 1$.

22. Find the residue at $z = 0$ of the function $f(z) = z \cos \frac{1}{z}$.

SECTION - III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4 marks each**.

23. Without evaluating the integral $\int_C \frac{dz}{z^4}$, determine the upper bound for its absolute value where C is the line segment from $z = 1$ to $z = i$.

24. State and prove Liouville's Theorem.

25. State and prove Cauchy's inequality.

26. Evaluate $\int_C \frac{z+2}{z} dz$ where C is the semicircle $z = 2e^{i\theta}$ where $0 \leq \theta \leq \pi$.

27. Prove that $\int_B \frac{dz}{z^2(z^2+9)} = 0$ where B consists of the circle $|z| = 2$ described in the positive direction together with the circle $|z| = 1$ described in the negative direction.

28. Find the residue of $f(z) = \frac{1}{(z^2 + a^2)^2}$ at $z = ai$.

29. Find the residue at $z = 0$ of $\frac{1+e^z}{z \cos z + \sin z}$.

30. Find the residue of $f(z) = \frac{1}{z(e^z - 1)}$ at the origin.

31. Use residues to evaluate the definite integral $\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta}$.



SECTION - IV

Answer **any 2** questions, from among the questions **32 to 35**. These questions carry **15 marks each**.

- 32 i) Show that $\int_C |z|^2 dz = -1+i$ where C is the square with vertices O (0, 0), A (1, 0), B (1, 1), C (0, 1). 10

- ii) Evaluate the integral $\int (x^2 - iy^2) dz$ where C is the parabola $y = 2x^2$ from (1, 2) to (2, 8). 5

33. State and prove Cauchy's integral formula. Use it to evaluate $\int_C \frac{z dz}{z^2 - 1}$ where C is positively oriented circle $|z| = 2$.

34. State and prove Cauchy residue theorem. Use this theorem to evaluate $\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz$ around the circle C given by $|z| = 2$.

35. Prove :

i) $\int_0^\infty \frac{2x^2 - 1}{x^4 + 5x^2 + 4} dx = \pi/4$. 8

ii) $\int_0^{2\pi} \frac{d\theta}{1 + a \sin \theta} = \frac{2\pi}{\sqrt{1 - a^2}}$ 7