

Reg. No. :

Name :

Fourth Semester B.Sc. Degree Examination, May 2021

First Degree Programme Under CBCSS

Mathematics

Core Course III

MM 1441 — ALGEBRA AND CALCULUS – II

(2015 –2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer the **first ten** questions are **compulsory**. They carry **1 mark each**.

1. Define a polynomial over a commutative ring R .
2. Find $p(1)$ for the polynomial $p(x) = x^3 + 1$ over $R = \mathbb{Z}/2\mathbb{Z}$.
3. Let $\text{Funct}(R, R)$ be the ring of functions from R to R . If R has n elements, how many elements are there in $\text{Funct}(R, R)$.
4. Show that $(x - 1)$ does not divide $x^3 - 2$.
5. Give an example of an irreducible polynomial of degree 3 in $\mathbb{Q}[x]$.
6. Let $f(x, y) = 3x^2\sqrt{y} - 1$. Find $f(ab, 9b)$.
7. Evaluate $\lim_{(x,y) \rightarrow (-1,2)} \frac{xy^3}{x+y}$.

8. Determine $f_y(1,3)$ for the function $f(x,y) = 2x^3y^2 + 2y + 4x$.

9. Evaluate $\int_0^1 xy^2 dy$.

10. Evaluate $\int_0^1 \int_0^1 y \sin x dy dx$.

SECTION – II

Answer **any eight** questions (11–22). **Each** question carries **2** marks.

11. Detach the coefficients and multiply in $\mathbb{F}_2[x]$ $(x^3 + x + 1)(x^4 + x^3 + 1)$.

12. Let $\mathbb{R} = \mathbb{Z}/4\mathbb{Z} = \{0, 1, 2, 3\}$. Show that $1 + 2x$ is a unit of $\mathbb{R}[x]$.

13. Find the quotient and remainder when $x^4 - 2x^2 - 1$ is divided by $x^2 + 3x - 1$ in $Q = [x]$.

14. Show that if $f(x)$ is a polynomial with coefficients in a field F and a is in F , then $f(a) = 0$ iff $(x - a)$ divides $f(x)$.

15. Show that if p is irreducible and f is any polynomial which is not divisible by p , then the greatest common divisor of p and f is 1.

16. Show that if $f(x) = x^2 + bx + c$ is a polynomial of degree 2 in $\mathbb{R}[x]$, then $f(x)$ is irreducible iff $b^2 - 4c < 0$.

17. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

18. If $f(\rho, \theta, \phi) = \rho^2 \cos \phi \sin \theta$, evaluate $f_\theta(\rho, \theta, \phi)$.

19. Given that $z = e^{xy}$, $x = 2u + v$, $y = \frac{v}{u}$. Find $\frac{\partial z}{\partial v}$.

20. Evaluate $\int_1^{3/2} \int_y^{3-y} y \, dx \, dy$.
21. Use a double integral to find the area of the region R enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$.
22. Find $\iiint_G 12xy^2 z^3 \, dV$ over the rectangular box G defined by the inequalities $-1 \leq x \leq 2, 0 \leq y \leq 3, 0 \leq z \leq 2$.

SECTION – III

Answer **any six** questions (23–31). **Each** question carries **4** marks.

23. Show that $\sum_{e|p-1} N(e) = p - 1$, where $N(e)$ is the number of elements of u_p , the group of units of $\frac{\mathbb{Z}}{p\mathbb{Z}}$ which have order e .
24. Show that a non zero polynomial of degree n in $F[x]$ has at most n distinct zeros in F .
25. Find the greatest common divisor in $\mathbb{Q}[x]$ of $(x^2 - 3x - 4)^3(x - 3)^2$ and $(x - 4)^3(x^2 - 3x - 4)^2$.
26. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^4 \sin(xy^3)$.
27. Show that when f is differentiable, a function of the form $z = f(xy)$ satisfies the equation $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$.
28. Locate all relative extrema and saddle points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

29. Evaluate the double integral $\iint_R y^2 x \, dA$ over the rectangle $R = \{(x, y) : -3 \leq x \leq 2, 0 \leq y \leq 1\}$.
30. Use a polar double integral to find the area enclosed by the three-petaled rose $r = \sin 3\theta$.
31. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

SECTION – IV

Answer **any two** questions (32–35). **Each** question carries **15** marks.

32. Let F be a field. Let f, g be two polynomials in $F[x]$ with $f \neq 0$. Show that there are polynomials q and r with $\deg r < \deg f$ such that $g = fq + r$. If also $g = fq_1 + r_1$ then show that $q = q_1$ and $r = r_1$.
33. Show that for any n , $\sum_{d|n} \phi(d) = n$.
34. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{2x^6 + y^2}$ does not exist.
35. Find the volume of the solid enclosed between the paraboloids $z = 5x^2 + 5y^2$ and $z = 6 - 7x^2 - y^2$.
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