



Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, December 2017
First Degree Programme under CBCSS
COMPLEMENTARY COURSE FOR CHEMISTRY AND
POLYMER CHEMISTRY
MM1331.2 : Mathematics – III
(Vector Analysis and Theory of Equations)
(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – 1

(Answer all questions. Each question carries 1 mark.)

1. State the fundamental theorem of Algebra.
2. Find the number of real roots of the equation $x^3 - x^2 + x - 1 = 0$.
3. If $f(x) = (x-2)^4 \phi(x)$ and $\phi(x)$ is not divisible by $(x-2)$, find the multiplicity of 2.
4. If α, β, γ are the roots of the equation $x^3 + 2x^2 - x + 1 = 0$ find the equation whose roots are $-\alpha, -\beta, -\gamma$.
5. If α, β are the roots of the equation $x^2 + 3x + 4 = 0$, find the equation whose roots are $1/\alpha$ and $1/\beta$.
6. Find the domain of $\overline{r(t)}$ if $\overline{r(t)} = (\sqrt{3t+1}, t^2)$. Also find $\overline{r(t_0)}$ if $t_0 = 1$.
7. Find $\lim_{t \rightarrow 3} (t^2 \vec{i} + 2t \vec{j})$.
8. Find $\overline{r'(t)}$ if $\overline{r(t)} = 4\vec{i} - \cos t \vec{j}$.



9. Find divergence of $\vec{F}$ if $\vec{F}(x, y, z) = x^2y\vec{i} + 2y^3z\vec{j} + 3z\vec{k}$.
10. Define a Solenoidal vector field. Give an example.

SECTION - 2

(Answer **any eight**. Each question carries 2 marks.)

11. Show that the equation $x^3 + 2x^2 + 7 = 0$ has at least two complex roots.
12. If 1, 2, 3 are the roots of the cubic equation $f(x) = 0$, find $f(x)$ and hence find the equation whose roots are 1, 1/2 and 1/3.
13. If 1 and 3 are the roots of $g(x) = 0$, find $g(x)$ and hence find the equation whose roots are 1/2 and 3/2.
14. Solve $4x^3 - 24x^2 + 23x + 18 = 0$ whose roots are in Arithmetic Progression.
15. Sketch the graph and a radius vector of $\vec{r}(t) = \cos t \vec{i} - \sin t \vec{j}$; $0 \leq t \leq 2\pi$.
16. Find $\vec{y}(t)$ if $\vec{y}'(t) = \vec{i} + e^t \vec{j}$; $\vec{y}(0) = 2\vec{i}$ and $\vec{y}'(0) = \vec{j}$.
17. Find $k(t)$ for the circular helix $x = a \cos t$, $y = a \sin t$, $z = ct$; $a > 0$.
18. Find the velocity and speed of a particle moving along the curve
 $\vec{r}(t) = t\vec{i} + \frac{t^2}{2}\vec{j} + \frac{t^3}{3}\vec{k}$ at $t = 1$.
19. Find the divergence and curl of the vector field $\vec{F}(x, y, z) = x^2y\vec{i} + 2y^3z\vec{j} + 3z\vec{k}$.
20. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that (a) $\text{Curl } \vec{r} = 0$ (b) $\text{div } \vec{r} = 3$.
21. Evaluate $\int_C x^2y dx + xdy$ along the triangular path joining (0, 0), (1, 0) and (1, 2) in the counter clockwise direction.
22. Let $\vec{F}(x, y) = e^y\vec{i} + xe^y\vec{j}$, prove that $\vec{F}$ is a conservative field in the entire plane.



SECTION-3

(Answer any six. Each question carries 4 marks.)

23. If α, β, γ are the roots of the equation $8x^3 - 4x^2 + 6x - 1 = 0$, find the equation whose roots are $\alpha + 1/2, \beta + 1/2$ and $\gamma + 1/2$.
24. Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ using bisection method, correct to four decimal places.
25. Solve the equation $x^3 - 5x + 3 = 0$, by Newton-Raphson method, correct to three decimal places.
26. Using the vector equation of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, find the curvature at the end points of the major and minor axes.
27. Suppose that a particle moves through 3-space so that its position vector at time t is $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$. Find
- The scalar tangential and normal components of acceleration at time t .
 - The scalar tangential and normal components of acceleration at time $t = 1$.
 - The vector tangential and normal components of acceleration at time $t = 1$.
28. Prove that $\text{curl } (\text{grad } \phi) = 0$, where ϕ is a scalar point function.
29. Find the work done by the force field $\vec{F}(x, y) = x^3y\vec{i} + (x - y)\vec{j}$ on a particle that moves along the parabola $y = x^2$ from $(-2, 4)$ to $(1, 1)$.
30. Let $\vec{F}(x, y) = 2xy^3\vec{i} + (1 + 3x^2y^2)\vec{j}$.
- Show that $\vec{F}$ is a conservative vector field in the entire x - y plane.
 - Find the scalar potential ϕ .
31. State Green's theorem. Using the theorem, evaluate $\oint_C x^2 y dx + x dy$ along the triangular path joining $(0, 0)$, $(1, 0)$ and $(1, 2)$ in the anticlockwise direction.



SECTION-4

(Answer any two. Each question carries 15 marks.)

32. Solve $x^4 - 2x^3 + 3x^2 + 4x - 1 = 0$, given that the product of two roots is unity.
33. Describe the Newton-Raphson method for finding the solution of the equation $f(x) = 0$. Solve the equation $x^4 + x^2 - 80 = 0$, by Newton-Raphson method, correct to three decimal places.
34. A shell fired from a cannon has a muzzle speed of 800ft/s. The barrel makes angle 45° with the horizontal, the barrel opening is assumed to be at the ground level. Find
- The parametric equations for the shell's trajectory
 - Maximum height reached
 - The horizontal distance travelled
 - Speed of the shell at the point of impact with the ground.
35. State Stoke's theorem. Verify the theorem for the vector field $\vec{F}(x, y, z) = 2z\vec{i} + 3x\vec{j} + 5y\vec{k}$ taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation, and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the x - y plane.