

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. (a) Maximize the profit for a firm, given that its total revenue function is $TR = 4000q - 33q^2$ and its total cost function is $TC = 2q^3 - 3q^2 + 400q + 5000$ where $q > 0$. **7**
- (b) The cost of building an office block, x floors high, comprises three components :
- Rs. 18 crores for the land,
 - Rs. 20 lakhs per floor,
 - Specialized costs of Rs. 2,00,000 x per floor. How many floors should the block contain if the average cost per floor is to be minimized. **8**
33. (a) If $y = \log_e(x + \sqrt{x^2 + 1})$, show that $(1 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$. **6**
- (b) A firm's unit capital and labour costs are respectively 2 and 4. The production function is $Q = 6KL + 2L^2$.
- If the total input costs are 200 units, what is the maximum possible output Q .
 - If the output is fixed at 1,200, what are the minimum input costs. **9**
34. A firm produces two goods G_1 and G_2 . The output of G_1 is denoted by Q_1 and its price by P_1 . Similarly for good G_2 . The production functions are $P_1 = 20 - Q_1 + 2Q_2$ and $P_2 = 10 + Q_1 - Q_2$. The total costs are given as $TC = 12Q_1 + Q_1Q_2 + 6Q_2$. The firm is contracted to produce a total of 20 units of goods of either type. What is the maximum profit possible?
35. (a) State Euler's theorem on homogeneous function and verify it for the function $f(x, y) = x^3 + 2x^2y + y^3$. **8**
- (b) Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log_e u$ where $\log_e u = \frac{(x^3 + y^3)}{(3x + 4y)}$. **7**

(2 × 15 = 30 Marks)

(Pages : 4)

R – 3094

Reg. No. :

Name :

Second Semester B.A. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1231.5 : MATHEMATICS FOR ECONOMICS – II

(2013 – 2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

- Find the second derivative of $\log_e(2x + 3)$.
- Find the approximate change in $y = 3x^2 + 4$ when x changes from $x = 2$ to $x = 2.1$.
- Determine whether the function $y = 3x^3 - 4x - 5$ is increasing or decreasing at $x = 1$.
- A company manufactures fuel tanks for cars. The total weekly cost of producing x tanks is given by $C(x) = 10,000 + 90x - 0.05x^2$. Find the marginal cost at a production level of 500 tanks per week.
- Use implicit differentiation to find if $\frac{dy}{dx}$ if $4x^2 - y^3 = 97$.
- Find the minimum point of the average cost function $AC(x) = \frac{25}{x} + 0.1x^2$.

7. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = x^2 + 2xy - y^2$.
8. Show that the curve $y = \log_e x$ is convex upward.
9. For $z = x^3y - 2y^4 + 3$, find $\frac{\partial^2 z}{\partial y \partial x}$.
10. Show that $f(x, y) = 3x^2y - y^3$ is a homogeneous function of degree 3.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carry **2** marks each.

11. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ by implicit differentiation when $x - y + 3xy = 2$.
12. If the average cost function for a good is $AC = \frac{24}{x} + 15 + 3x$, find an expression for the total cost function. What are the fixed costs in this case? Write down an expression for the marginal cost function.
13. Find the maximum and minimum values of the function $f(x) = 3x^2 - 6x + 5$ over $[0, 3]$.
14. Show that the function $f(x) = x^4$ does not have an inflection point at $x = 0$.
15. A company manufactures and sell x transistor radios per week. If the weekly cost and revenue equations are $C(x) = 5,000 + 2x$, $R(x) = 10x - \frac{x^2}{1,000}$, $0 \leq x \leq 8,000$ then find the approximate change in revenue and profit if production is increased from 2,000 to 2,010 units per week.
16. Find the marginal cost and average cost function associated with the total cost function $TC = 3x^2 + 7x + 12$.
17. Find the total derivative of $u = x^2 + y^2$ with respect to t where $x = t^2$ and $y = t^2 + 1$.
18. Verify Young's theorem for the function $f(x, y) = x^3 + x^4y^2 + y^3$.
19. Estimate the change in $z = x^2 + 3y$ when x increases from 5 to 5.01 and y decreases from 8 to 7.98.

20. Find the first and second order differential of the function $u = x^2y^2$.
21. Find the extreme value of the function $u = x^3 + x^2 - xy + y^2 + 4$.
22. Verify the homogeneity of the function $f(x, y) = \frac{(x^2 - y^2)^3}{x + y}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Find the first and second derivatives of $y = \frac{e^x}{x}$.
24. A firm faces the demand schedule $p = 184 - 4q$ and the total cost function $TC = q^3 - 21q^2 + 160q + 40$. What output will maximize the profit?
25. Find the critical values of f , the intervals on which f is increasing, and those on which f is decreasing for $f(x) = 8 \log_e x - x^2$.
26. Find the inflection points of the curve $y = \log_e(x^2 - 4x + 5)$.
27. Show that if marginal cost is greater than the average cost then average cost is increasing.
28. Find $\frac{dz}{dx}$ and the values of x for which $\frac{dz}{dx} = 0$, given that $z = 4x^2y^3$, where $y = x^2 + 3$.
29. Optimize the function $z = 4x^2 + 3xy + 6y^2$ subject to the constraint $x + y = 56$.
30. Find the first and second order partial derivatives of $z = (x^2 + 2y)^4$.
31. If x increases at the rate 2 cm/sec at the instant when $x = 3$ cm and $y = 1$ cm, at what rate must y be changing in order that the function $2xy - 3x^2y$ shall be neither increasing nor decreasing.

(6 × 4 = 24 Marks)