



Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, July 2017
First Degree Programme under CBCSS
COMPLEMENTARY COURSE FOR PHYSICS
MM 1231.1 : Mathematics – II : Integration and Vectors
(2013 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They carry 1 mark each.

1. Evaluate $\int \frac{\sin x}{\cos^2 x} dx$.
2. Find the average value of the function $f(x) = x^2$ over the interval $[-1, 2]$.
3. A particle moves with a velocity of $v(t) = t^3 - 3t^2 + 2t$ m/s along an s-axis. Find the displacement of the particle during the time interval $0 \leq t \leq 3$.
4. Show that the vectors $\vec{u} = 4\mathbf{i} + \mathbf{j} + 6\mathbf{k}$ and $\vec{v} = -3\mathbf{i} + 2\mathbf{k}$ are orthogonal.
5. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be three non-zero vectors in 3-space lying in the same plane. What is the value of the scalar triple product $\vec{u} \cdot (\vec{v} \times \vec{w})$?
6. Find $\lim_{t \rightarrow +\infty} \left\langle \frac{t^2+1}{3t^2+2}, \frac{1}{t} \right\rangle$.
7. Find $\nabla \left(\frac{x+y}{x-y} \right)$.
8. Evaluate $\int_C (x-y) dx$ where C be the curve represented by the equations $x = 2t$, $y = 3t^2$, $0 \leq t \leq 1$.



9. Consider the surface integral $\iint_{\sigma} f(x, y, z) \, dS$ where σ is the graph of a function $x = g(y, z)$ with continuous first order partial derivatives. What should replace dS to evaluate the integral?
10. State Gauss's Law for electric fields.

SECTION - II

Answer **any 8** questions from among the questions **11 to 22**. These questions carry **2 marks each**.

11. Find $\frac{dy}{dx}$ if $y = \int_1^x \sqrt{1 + \cos^2 t} \, dt$.
12. Find the area of the region enclosed by the curves $y = x^2$ and $y = 4x$.
13. Suppose that the velocity function of a particle moving along a coordinate line is $v(t) = 3t^3 + 2$. Find the average velocity of the particle over the time interval $1 \leq t \leq 4$.
14. Find the volume under the plane $z = 2x + y$ and over the rectangle $R = \{(x, y) : 3 \leq x \leq 5, 1 \leq y \leq 2\}$.
15. Let $\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}$, $\vec{v} = \vec{j} + 7\vec{k}$, $\vec{w} = \vec{i} + 4\vec{j} + 5\vec{k}$, find $\vec{u} \times (\vec{v} \times \vec{w})$.
16. Find the unit tangent vector to the curve $\vec{r}(t) = 4\cos t \, \vec{i} + 4\sin t \, \vec{j} + t\vec{k}$ at the point $t = \frac{\pi}{2}$.
17. Find $\text{curl } \vec{F}$ where $\vec{F}(x, y, z) = e^{xy} \vec{i} - \cos y \vec{j} + \sin^2 z \vec{k}$.
18. Find $\nabla \cdot (\vec{F} \times \vec{G})$ where $\vec{F}(x, y, z) = 2x\vec{i} + \vec{j} + 4y\vec{k}$ and $\vec{G}(x, y, z) = x\vec{i} + y\vec{j} - z\vec{k}$.
19. State Stoke's theorem, including all of the required hypotheses.
20. Verify whether $\vec{F}(x, y, z) = \sin x \, \vec{i} + z \, \vec{j} + y \, \vec{k}$ is a conservative field.



21. Evaluate $\int y \, dx + z \, dy - x \, dz$ along the line segment from $(0, 0, 0)$ to $(1, 1, 1)$.
22. Find a nonzero function $h(x)$ for which $\vec{F}(x, y) = h(x) \sin y \, \mathbf{i} + h(x) \cos y \, \mathbf{j}$ is conservative.

SECTION - III

Answer **any 6** questions from among the questions **23 to 31**. These questions carry **4** marks **each**.

23. At each point (x, y) on the curve, y satisfies the condition $\frac{d^2y}{dx^2} = 6x$. Also the line $y = 5 - 3x$ is tangent to the curve at the point where $x = 1$. Find the equation to the curve that satisfies the given conditions.
24. Evaluate the triple integral $\iiint_G y \, dV$, where G is the solid enclosed by the plane $z = y$, the xy -plane and the parabolic cylinder $y = 1 - x^2$.
25. Using polar coordinates, evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} \, dy \, dx$.
26. Find the curvature of the circular helix $x = a \cos t$, $y = a \sin t$, $z = ct$ where $a > 0$.
27. Find the volume of the parallelepiped with adjacent edges $\vec{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\vec{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\vec{w} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$.
28. Suppose that a particle moves through 3-space so that its position vector at time t is $\vec{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the scalar tangential and normal components of acceleration at time t .
29. Find the work done by the force field $\vec{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$ on a particle that moves along the curve $C: \vec{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \leq t \leq 1$.
30. Evaluate $\oint_C y \tan^2 x \, dx + \tan x \, dy$ using Green's theorem where C is the circle $x^2 + (y + 1)^2 = 1$.
31. Using line integral, find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.



SECTION - IV

Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each**.

32. a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about y-axis.
- b) Use a triple integral to find the volume of the wedge in the first octant that is cut from the solid cylinder $y^2 + z^2 \leq 1$ by the planes $y = x$ and $x = 0$.
33. a) Prove that $\text{div}(\text{curl } \vec{F}) = 0$.
- b) Find the directional derivative of the function $f(x, y, z) = x^3y^2z^5 - 2xz + yz + 3x$ at $P(-1, -2, 1)$ in the direction of the negative z-axis.
34. a) Evaluate $\int_{(-1,2)}^{(1,3)} y^2 dx + 2xy dy$.
- b) Consider the vector field given by $\vec{F}(x, y, z) = (z + \sin x) \mathbf{i} + (x + y^2) \mathbf{j} + (y + e^z) \mathbf{k}$. Use Stoke's theorem to find the circulation around C where C is the intersection of the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z = \sqrt{x^2 + y^2}$ with counterclockwise orientation looking down the positive z-axis.
35. Use the Divergence theorem to find the outward flux of the vector field $\vec{F}(x, y, z) = x^3 \mathbf{i} + y^3 \mathbf{j} + z^3 \mathbf{k}$ across the surface of the region that is enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$.
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