



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2019

First Degree Programme under CBCSS

MATHEMATICS

Core Course

MM 1642 : Linear Algebra
(2013 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are compulsory. They carry 1 mark each.

1. When do we say that two matrices are equal ?
2. Find $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$
3. What is a consistant system of equations ?
4. Define orthogonal matrices.
5. Define kernel of a linear transformation.
6. What is a linear isomorphism ?
7. Define ordered basis.
8. If $f = (1 \ 6 \ 4 \ 3 \ 5 \ 2)$ and $g = (2 \ 6 \ 5 \ 3 \ 1 \ 4)$, find $g \circ f$.
9. What is a transposition ?
10. Write an example of a tridiagonal matrix.

SECTION – II

Answer any 8 questions from among the questions 11 to 22. These questions carry 2 marks each.

11. Write out the 3×3 matrix whose entries are given by $x_{ij} = i + j$.
12. Prove that the addition of matrices is commutative.

P.T.O.



13. If A is orthogonal, prove that A' is also orthogonal.
14. Prove that every non-zero matrix A can be transformed to a Hermite matrix by means of elementary row operations.
15. Define left inverse and right inverse of a matrix A .
16. If A and B are invertible matrices, prove that $(AB)^{-1} = B^{-1}A^{-1}$.
17. Prove that a 2×2 matrix is orthogonal if and only if it is of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ or $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.
18. Determine which of the following mappings are linear?
a) $f(x, y, z) = (y, z, 0)$ b) $g(x, y, z) = (|x|, z, 0)$
19. Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$, be given by $f(x, y, z) = (2x - 3y + z, 3x - 2y)$. Find the matrix off relative to the natural ordered basis of $\mathbb{R}^3$ and $\mathbb{R}^2$.
20. Prove that transition matrices are invertible.
21. Define determinantal mapping.
22. Prove that a scalar λ is an eigenvalue of an $n \times n$ matrix A if and only if $\det(A - \lambda I_n) = 0$.

SECTION - III

Answer **any 6** questions from among the questions **23 to 31**. These questions carry **4 marks each**.

23. Write a short note on an application of matrices.

24. Explain the effect of left multiplication by the matrix $\begin{bmatrix} 0 & \alpha & 1 \\ 0 & \beta & 0 \\ 1 & \gamma & 0 \end{bmatrix}$.

25. Reduce to row-echelon form of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 5 & 5 & 8 \end{bmatrix}$.

26. Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 4 \end{bmatrix}$.



27. If $f: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ is defined as $f(a, b, c, d) = (a + b, b - c, a + d)$, find a basis for $\text{Im } f$.
28. Prove that a linear mapping is completely and uniquely determined by its action on basis.
29. Consider the linear mapping $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by $f(x, y) = (x + 2y, 2x - y, -x)$. Determine the matrix of f
- relative to the natural ordered basis
 - relative to the basis $\{(0, 1), (1, 1)\}$ and $\{(0, 0, 1), (0, 1, 1), (1, 1, 1)\}$.

30. Find the determinant of $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 2 & 0 & 2 \\ 1 & 2 & 3 & 0 \\ 5 & 3 & 2 & -1 \end{bmatrix}$.

31. Prove that every connected graph has at least one spanning tree.

SECTION - IV

Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each**.

32. Given $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$, find P and Q such that $PAQ = N$,

where N is the normal form of A .

33. State and prove dimension theorem.
34. Prove that the rank of a linear mapping f is same as the rank of any matrix that represents f .
35. Find the limiting value of the sequence $2, 2 + \frac{1}{2}, 2 + \frac{1}{2 + \frac{1}{2}}, 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, \dots$.
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