

33. (a) Solve the equation $z^6 - z^5 + 4z^4 - 6z^3 + 2z^2 - 8z + 8 = 0$.

- (b) Plane polar co-ordinates, ρ and ϕ and the Cartesian co-ordinates x and y are related by the expressions

$x = \rho \cos \phi$ and $y = \rho \sin \phi$. An arbitrary function $f(x, y)$ can be re-expressed as a function $g(\rho, \phi)$. Transform the expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ into one in ρ and ϕ .

34. (a) Find the mass of the tetrahedron bounded by the 3 co-ordinate surfaces and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, if the density is given by $\rho(x, y, z) = \rho_0 \left(1 + \frac{x}{a}\right)$.

- (b) Find the area of the region enclosed by the parabola $y = x^2$ and the line $y = x + 2$

35. (a) Express the vector field $a = yz\bar{i} - y\bar{j} + xz^2\bar{k}$ in cylindrical polar co-ordinates and hence calculate its divergence.

- (b) Show that the acceleration of a particle travelling along a trajectory $r(t)$ is given by $a(t) = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}$, where v is the speed of the particle, $\hat{t}$ is the unit tangent to the trajectory, $\hat{n}$ is its principal normal and ρ is its radius of curvature.

(2 × 15 = 30 Marks)

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – CALCULUS WITH APPLICATIONS
IN PHYSICS – II

(2018 – 2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION I

All 10 questions are compulsory. Each question carries 1 mark.

1. Represent $1 + 2i$ in Argand diagram.
2. Find $(3 + 4i) + (18 + 10i)$.
3. State De-Moivre's theorem.
4. Find $\frac{\partial f}{\partial x}$ if $f(x, y) = 2x + y$.
5. Write Taylor series expansion for functions of 2 variables.
6. Evaluate $\int_0^2 \int_{-1}^1 (x - y) dy dx$.
7. Write down the formula for finding the area of a closed bounded region R .
8. State Pappus theorem.

9. Find velocity of a particle if $r(t) = (t+1)\bar{i} + (t^2-1)\bar{j} + 2t\bar{k}$.
10. Find the divergence of the vector field $a = x^2y^2\bar{i} + y^2z^2\bar{j} + x^2z^2\bar{k}$.

(10 × 1 = 10 Marks)

SECTION II

Answer **any eight** questions. Each question carries **2** marks.

11. Express z in the form of $x + iy$ where $z = \frac{3-2i}{-1+4i}$.
12. Prove that $\cos 4\theta = \cos^4 \theta - 6\cos^2 \theta \sin^2 \theta + \sin^4 \theta$.
13. Prove that $\frac{d}{dx}(\cosh x) = \sinh x$.
14. Verify that $|z_1 z_2| = |z_1| |z_2|$, where $z_1 = 3 + 2i$ and $z_2 = -1 - 4i$.
15. Find the total differential of the function $f(x, y) = ye^{x+y}$.
16. Check whether the differential equation $(y^2 - 2x)dx + (2xy + 1)dy = 0$ is exact or not.
17. Given that $x(u) = 1 + au$ and $y(u) = bu^3$, find the rate of change of $f(x, y) = xe^{-y}$ with respect to u .
18. Define the partial derivative of $f(x, y)$ with respect to y .
19. Calculate $\iint_R f(x, y) dx dy$ if $f(x, y) = 100 - 6x^2y$ and $R: 0 \leq x \leq 2$ and $-1 \leq y \leq 1$.
20. Write any two properties of 'Jacobian'.
21. Find the curl of the vector field $x^2z\bar{i} - 2xz\bar{j} + yz\bar{k}$.
22. Find the Laplacian of the scalar field $\phi = xy^2z^3$.

(8 × 2 = 16 Marks)

SECTION III

Answer **any six** questions. Each question carries **4** marks.

23. Express $\sin 5\theta$ in terms of $\sin \theta$.
24. Prove that $\cosh^{-1} x = \log \left[x + \sqrt{x^2 - 1} \right]$.
25. Find the solution to the equation $z^3 = 1$.
26. Find the Taylor series expansion upto quadratic terms in $(x-2)$ and $(y-3)$ of $f(x, y) = ye^{xy}$ about the point $x = 2$ and $y = 3$.
27. Find all the second order partial derivatives of the function $f(x, y) = 2x^2y + x^2y^6 - 1$.
28. Find the centre of mass of the solid hemisphere bounded by the surface $x^2 + y^2 + z^2 = a^2$ and the XY plane assuming that it has a uniform density?
29. Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$.
30. A semicircular uniform lamina is freely suspended from one of its corners. Show that its straight edge makes an angle of 23.0° with the vertical.
31. For the function $\phi = x^2y + yz$ at the point $(1, 2, -1)$, find its rate of change with distance in the direction $a = \bar{i} + 2\bar{j} + 3\bar{k}$. At the same point, what is the greatest possible rate of change with distance and in which direction does it occur.

(6 × 4 = 24 Marks)

SECTION IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) By Integrating $e^{(a+ib)x}$, find $\int e^{ax} \cos bx dx$ and $\int e^{ax} \sin bx dx$.
- (b) Prove that $\sin^6 \theta = -\frac{1}{2^5} [\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10]$.