

Reg. No. : .....

Name : .....

**Fourth Semester B.Sc. Degree Examination, May 2021**

**First Degree Programme Under CBCSS**

**Mathematics**

**Complementary Course for Chemistry/Polymer Chemistry**

**MM 1431.2 : MATHEMATICS IV – DIFFERENTIAL EQUATIONS, VECTOR  
CALCULUS AND ABSTRACT ALGEBRA**

**(2019 Admission Regular)**

Time : 3 Hours

Max. Marks : 80

I. All the **first ten** questions are compulsory. **Each** question **1** mark.

1. Solve  $\frac{dy}{dx} - 2x = 0$ .
2. Write the standard form of Bernoulli's equation.
3. Give an example of a non-linear differential equation.
4. True / False:  $W(y_1, y_2) = 0$  implies  $y_1$  and  $y_2$  are dependent
5. Find  $\text{div } F$ , if  $F = xi + yj$ .
6. Give an example of a one- sided surface.
7. State Divergence Theorem.

8. Give an example of a non-abelian group.
9. Define order of a group.
10. True/False : If  $n_\mu$  is the dimension of the  $\mu^{\text{th}}$  irrep of a group  $G$ , then  $\sum_\mu n_\mu^2 = g$ , where  $g$  is the order of  $G$ .

(10 × 1 = 10 Marks)

- II. Answer **any eight** questions from among the questions 11 to 26. Each carries 2 marks.

11. Find the order and degree of the Differential Equation  $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{2}} = 0$ .
12. Solve  $\frac{dy}{dx} = xy - x$ .
13. Find the general solution of  $y = p + p^2x$ .
14. Find the complementary function of  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3e^x$ .
15. Write the standard form of Legendre's linear equation and Euler linear equation
16. Solve  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2 = 0$ .
17. Find  $\text{div } F$ , where for  $F = 3xy \mathbf{i} + 4yx^2 \mathbf{j}$ .
18. Write the integral form for grad, div and curl.
19. Evaluate the line integral  $I = \int_C 3x^2 - 2y + z$ , where  $C$  is the line segment joining  $(0,0,0)$  and  $(1,1,1)$
20. Find an expression for the angular momentum of a solid body rotating with angular velocity  $\omega$  about an axis through the origin.
21. Define an equivalence relation.
22. Define representation of a group  $G$  with an example.
23. Define  $S_n$ . Find the order of  $S_n$ .

24. Define homomorphism of groups. Define Kernel of the homomorphism
25. State the Cancellation laws in group theory
26. Find the order of  $\phi = (3) (1\ 4\ 6) (2\ 5)$

(8 × 2 = 16 Marks)

III. Answer **any six** questions from among the questions 27 to 38. Each carries 4 marks.

27. Solve  $x \frac{dy}{dx} + 3x + y = 0$ .
28. Solves  $\frac{dy}{dx} = \frac{y}{x} + \tan^{-1} \frac{y}{x}$ .
29. Find the general solution of the equation  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = e^x$ .
30. Solve  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 4y = 0$ .
31. Calculate  $\nabla \times F$  for  $F = xyz\ i + 3x^2y\ j + (xz^2 - y^2z)\ k$ .
32. Find the volume enclosed between a sphere of radius  $a$  centered in the origin and a circular cone of half angle  $\alpha$  with its vertices at the origin
33. Evaluate the line integral  $I = \oint_C x\ dy$ , where  $C$  is the circle in the  $xy$ -plane defined by  $x^2 + y^2 = a^2, z = 0$ .
34. Using Green's theorem, find the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
35. Prove that the inverse of an element in a group is unique.
36. Prove that conjugacy is an equivalence relation.
37. Prove that two cosets are either disjoint or identical.
38. Draw the table of products for the elements of the group  $\{1, 3, 5, 7\}$  under multiplication (mod 8).

(6 × 4 = 24 Marks)

IV. Answer **any two** questions from among the questions **39** to **44**. Each carries **15** marks.

39. (a) Solve  $\frac{dy}{dx} = \frac{2x - 5y + 3}{2x + 4y - 6}$ .

(b) Solve  $\frac{dy}{dx} + \frac{y}{x} = 2x^3 y^4$ .

40. (a) Solve  $\frac{d^2 y}{dx^2} + 4y = x^2 \sin 2x$ .

(b) Solve  $\frac{d^2 y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{-x}$  subject to the boundary conditions  $y(0) = 2, y'(0) = 1$ .

41. Evaluate the line integral  $I = \int_C a \cdot dr$ , where  $a = (x + y)i + (y - x)j$ , along each of the paths in the  $xy$  plane, namely

(a) the parabola

(b) the curve

(c) the line

42. Show that  $Q = (2xyz)i + (x^2z)j + (x^2y)k$  is a conservative field. Find its potential function. Evaluate  $\int_{(1,1,1)}^{(-1,2,2)} Q \cdot dr$ .

43. Explain the elements of a group of two dimensional operations which transform an equilateral triangle into itself, by drawing a sequence of model triangles. Draw the group table.

44. Find the group  $G$  generated under matrix multiplication by the matrices  $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ . Determine its proper subgroups and verify for each of them that its cosets exhaust  $G$ .

(2 × 15 = 30 Marks)