

SECTION – D

Answer any **two** questions. These question carries **15** marks each.

32. (a) If $z = x^y + \sin xy$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.
- (b) The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$. Find the temperature of the two hottest points on the circle.
33. Find the Maclaurin's series for
- (a) $\ln(2 + x)^3$
- (b) $(x^2 + 9)^{-1/2}$
- (c) $\exp \sin x$
34. (a) Find the element of area on the surface of a sphere of radius a and hence calculate the total surface area of the sphere.
- (b) For the function $\phi = x^2y + yz$ at the point $(1, 2, -1)$, find the rate of change with distance in the direction $a = i + 2j + 3k$. At this same point, what is the greatest possible rate of change with distance and in which direction does it occur?
35. (a) Evaluate the double integral $I = \iint (a + \sqrt{x^2 + y^2}) dx dy$, over the region R bounded by the circle $x^2 + y^2 = a^2$.
- (b) Find the mass of the tetrahedron bounded by the 3 co-ordinate surfaces and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, if its density is given by $\rho(x, y, z) = \rho_0(1 + \frac{x}{a})$.

(2 × 15 = 30 Marks)

(Pages : 4)

R – 3131

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry / Polymer Chemistry

MM 1231.2 : MATHEMATICS – II

CALCULUS WITH APPLICATIONS IN CHEMISTRY – II

(2018 – 2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Check whether $x dy + y dx$ is exact.
2. Find the first partial derivatives of $x^2 + y^2 + 4$.
3. Define stationary point.
4. Sum the integers between 1 and 500 inclusive.
5. Check whether $\sum_{n=0}^{\infty} \frac{1}{(n!)^2}$ converges or not.
6. State Comparison test to check the convergence.
7. Find the Gradient of the scalar field $\phi = xy^2 z^3$.
8. Define the Divergence of a vector field.

9. Evaluate $\int_0^3 \int_0^1 xy \, dy \, dx$.

10. State Pappu's first Theorem.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. These questions carry **2** marks each.

11. Find the first and second partial derivatives of the function $f(x, y) = x^3y$.
12. Find and evaluate maxima, minima and saddle points of a function $f(x, y) = xy(x^2 + y^2 - 1)$.
13. Find the total differential of the function $f(x, y) = ye^{x+y}$.
14. Determine whether the series $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{25} + \dots$ converges.
15. Find sum S_N of the first N terms of $\sum_0^\infty (-2)^n$ and determine whether the series converges, diverges or oscillates.
16. Expand $\sin x$ as Taylor series about $x = \frac{\pi}{2}$.
17. Find the Laplacian of the scalar field $\phi = x^2y^2z^2$.
18. Find the curl of the vector field $v = xyz\mathbf{i} + 3x^2y\mathbf{j} + (xz^2 - y^2z)\mathbf{k}$ at the point $(2, -1, 1)$.
19. If $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, show that $\text{grad } \frac{1}{r} = -\frac{\vec{r}}{r^3}$.
20. Change the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$.
21. Evaluate $\int_0^a \int_0^a \int_0^a (yz + zx + xy) \, dx \, dy \, dz$.
22. Express the volume of a region as double integral and triple integral.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. These question carry **4** marks each.

23. Find $\frac{\partial^2 f}{\partial x \partial y}$ where $f(x, y) = \frac{xy}{x+y}$.
24. Find the first and second order partial derivatives of the function $f(x, y) = x^2y + \cos y + y \sin x$.
25. Find $\frac{dz}{dt}$ where $z = x^2 + y^2$; $x = \cos t$, $y = \sin t$.
26. Determine whether the following series are absolutely convergent, convergent or oscillatory.
 - (a) $\sum_{n=1}^\infty \frac{(-1)^n}{n^{\frac{5}{2}}}$
 - (b) $\sum_{n=1}^\infty \frac{(-1)^n}{n} (2n+1)$
27. Sum the series $\sum_{n=1}^N (n+1)(n+3)$.
28. Show that $\text{Div. (Curl } V) = 0$.
29. Show that the vector field $v = (\sin y + z)\mathbf{i} + (x \cos y - z)\mathbf{j} + (x - y)\mathbf{k}$ is irrotational.
30. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
31. Change the order of integration in $\int_0^{12-x} \int_{x^2} xy \, dy \, dx$ and hence evaluate the same.

(6 × 4 = 24 Marks)