

29. Find $\iint_R (2x - y^2) dA$ over the triangular region R enclosed between the lines $y = -x + 1$, $y = x + 1$ and $y = 3$.
30. Compute the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0, 1] \times [0, 2]$.
31. Evaluate $\int_1^3 \int_2^4 (40 - 2xy) dy dx$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

32. (a) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
- (b) Evaluate : $\int e^x \cos x dx$
33. (a) Calculate the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y -axis.
- (b) Use cylindrical shells to find the volume of the solid generated when the region under $y = x^2$ over the interval $[0, 2]$ is revolved about the line $y = -1$.
34. Sketch the graph $r = a(1 - \cos \theta)$ in polar coordinates, assuming a to be a positive constant.
35. (a) A heat – seeking, particle is located at the point $(2, 3)$ on a flat metal plate whose temperature at (x, y) is $T(x, y) = 10 - 8x^2 - 2y^2$. Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.
- (b) Evaluate : $\int_0^1 \int_{-x}^{x^2} y^2 x dy dx$.
- (c) Estimate : $\int_0^{\pi/3} \int_0^{\cos y} x \sin y dx dy$.

(2 × 15 = 30 Marks)

(Pages : 4)

R – 3132

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry / Polymer chemistry

MM 1231.2 : MATHEMATICS II – INTEGRAL CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions.

1. Give an example of a proper rational function.
2. Evaluate: $\int \sin 7x \cos 3x dx$.
3. Write the formula for integration by parts.
4. What is a surface of revolution?
5. Write the second – order model of the period T of a simple pendulum.
6. Find the parametric equations for the surface generated by revolving the curve $y = \frac{1}{x}$ about the x – axis.

7. Write the conversion formulas for spherical coordinate system to rectangular coordinate system.
8. If $r(t) = t^2 i + e^t j - (2 \cos \pi t) k$, find $r'(t)$.
9. Define the gradient of $f(x, y, z)$.
10. Evaluate $\int_0^2 (2ti + 3t^2 j) dt$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions.

11. Evaluate : $\int \tan^2 x \sec x dx$.
12. Solve the initial value problem $\frac{dy}{dx} = \cos x$, $y(0) = 1$.
13. Using integration by parts, compute : $\int x^2 \sqrt{x-1} dx$.
14. Estimate : $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$.
15. Compute : $\int_0^2 x(x^2 + 1)^3 dx$
16. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y – axis.
17. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.
18. Use polar coordinates to compute $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} dy dx$.

19. Find the partial derivatives of $r = ui + vj + (4 - u^2 - v^2)k$.
20. Find the polar coordinates of the point whose rectangular coordinates are $(-2, -2\sqrt{3})$.
21. Let $f(x, y) = xy$. Find $D_u f(1, 2)$ for the unit vector $u = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$.
22. Find the directional derivative of $f(x, y, z) = x^2 y - yz^3 + z$ at the point $(1, -2, 0)$ in the direction of the vector $a = 2i + j - 2k$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

23. Separate $\frac{2x+4}{x^3-2x^2}$ into partial fractions.
24. Evaluate : $\int_0^1 \tan^{-1} x dx$.
25. Find the total area between the curve $y = 1 - x^2$ and the x – axis over the interval $[0, 2]$.
26. Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x – axis.
27. Prove : $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
28. Estimate $\iint_A \sin \theta dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos \theta)$.