



Reg. No. :

Name :

First Semester B.Sc. Degree Examination, February 2018**First Degree Programme under CBCSS****Complementary Course I for Physics****MM 1131.1 : MATHEMATICS – I : DIFFERENTIATION AND ANALYTIC****GEOMETRY****(2013 Admission)**

Time : 3 Hours

Max. Marks : 80

SECTION – I**All the first ten questions are compulsory. They carry 1 mark each.**

1. Define the jump of a function.
2. Write the parametric equation of a cycloid.
3. State Rolle's theorem.
4. Evaluate $\int_0^e \frac{dx}{x + e}$.
5. State Euler's theorem on Homogenous function.
6. State the rotation equations in Cartesian coordinates.
7. State rotation equations in Cartesian coordinates.
8. State reflection property of a Hyperbola.
9. Write the discriminant of the quadratic equation $x^2 - xy + y^2 - 2 = 0$ and identify the graph.
10. Define hyperbolic cosine function and sine function.



SECTION - II

Answer **any 8** questions from among the questions **11 to 22**. These question carry **2 marks each**.

11. Explain direct proportion with an example.

12. Draw the graph of the function $y = |x - 5| + 1$.

13. Find $\lim_{t \rightarrow 1} \frac{t^3 + t^2 - 5t + 3}{t^3 - 3t + 2}$.

14. If $g(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & \text{if } x \neq 3 \\ 6, & \text{if } x = 3 \end{cases}$. Is $g(x)$ continuous at $x = 3$?

15. Find the average rate of change of $y = \frac{1}{x^2}$ with respect to x .

16. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

17. Find the horizontal asymptote to the curve $y = \frac{5x + 2}{x}$.

18. Find the equation of the tangent line to the curve $y = \frac{1-x}{1+x}$ at $x = 2$.

19. Let $S(t) = t^3 - 6t^2$, be the position function of a particle moving along s -axis, where S is in meters and t in seconds. Find the instantaneous acceleration $a(t)$ and show the graph of acceleration versus time.

20. Find a formula for $f'(x)$, where $f(x) = \sqrt[5]{4x + 2}$.

21. Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ if $x = u + 2v^2$, $y = 2u^2 - v$.

22. Find $\lim_{x \rightarrow 1} (2 - x)^{\tan\left(\frac{\pi}{2}x\right)}$.



SECTION - III

Answer any 6 questions from among 23 to 31. These questions carry 4 marks each.

23. A garden is to be laid out in a rectangular area protected by wire fence. What is the largest possible area of the garden if only 100 running feet of wire is available for the fence.
24. a) Let 'f' be a function that is continuous on a closed interval $[a, b]$ and differentiable in (a, b) , if $f'(x) = 0$ for every value of x in (a, b) prove that f is constant in $[a, b]$.
- b) State and prove constant difference theorem.
25. Find a binomial series for $\frac{1}{\sqrt{1+x}}$.
26. Find the interval of convergence of the power series $1 + (x-2) + (x-2)^2 + \dots + (x-2)^k + \dots$
27. Let $F(x) = \int_0^x \frac{\cos t}{t^2 + 3} dt$, find $F(0)$, $F'(0)$ and $F''(0)$.
28. If $u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2} = \frac{\partial^2 u}{\partial y \partial x}$.
29. Locate all relative extrema and saddle point of $f(x, y) = 4xy - x^4 - y^4$.
30. Sketch the graph of the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$, showing its foci.
31. A line tangent to the hyperbola $4x^2 - y^2 = 36$ intersects the y -axis at the point $(4, 0)$. Find the point of tangency.



SECTION - IV

Answer any 2 questions from among the questions 32 to 35. These questions carry 15 marks each.

32. a) Find all points on the curve $x^2 - y^2 = 1$ that is closest to $(0, 2)$.
- b) Using Taylor's theorem expand $\tan(\pi/4 + x)$.
- c) A camera is mounted at a point 3000 ft from the base of a rocket launching pad. If the rocket is rising vertically at 880 ft/s when it is 4000 ft above the launching pad, how fast must the camera elevation angle change at that instant to keep the camera aimed at the rocket.
33. a) If w is a function of x and y and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ then prove that
- $$\frac{\partial w}{\partial u} - \frac{\partial w}{\partial v} = x \frac{\partial w}{\partial x} - y \frac{\partial w}{\partial y}.$$
- b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$.
- c) If $v = f\left(\frac{x}{z}, \frac{y}{z}\right)$, using appropriate chain rule show that $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + z \frac{\partial v}{\partial z} = 0$.
34. a) Let $y = x^3$, find dy and Δy at $x = 1$ with $dx = \Delta x = 1$ and sketch the graph of $y = x^3$, showing dy and Δy .
- b) Identify and sketch the curve $xy = 1$.
35. a) Prove that the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_0, y_0) is $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1$.
- b) Sketch the graph of $r = \frac{4}{2 + 3 \cos \theta}$ in polar coordinates.