

SECTION – D

Answer any **two** questions. These question carry **15** marks each.

32. (a) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^3 - 3x^2 + 2x$ over $[0, 1]$ is revolved about the y-axis.
- (b) Find the area enclosed by the curve $x^{2/3} + y^{2/3} = a^{2/3}$.
33. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.
- (b) Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x-axis.
34. (a) Use the relationship $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$ to find the first four nonzero terms in the Maclaurin series for $\sin^{-1} x$.
- (b) Evaluate $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$ in two ways; using L'Hopital's rule and by replacing $\sin x$ by its Maclaurin's series.
35. (a) Use triple integration to find the volume of the tetrahedron in the first octant bounded by the coordinate surfaces $x = 0$, $y = 0$, and $z = 0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ ($a > 0$, $b > 0$, $c > 0$).
- (b) Use triple integration in cylindrical coordinates to evaluate $\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} x^2 dz dy dx$.

(2 × 15 = 30 Marks)

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – APPLICATIONS OF CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first **Ten** questions are compulsory. They carry **1** mark each.

1. Show that the function $f(x) = x^3 + 6x^2 + 12x - 9$ is increasing for all values of x .
2. Define inflection point of a function.
3. Determine where the graph of the function $f(x) = x + 2 \sin x$ is concave up.
4. Define critical point of a function.
5. State Rolle's theorem.
6. Evaluate $\int_0^1 \int_0^2 (x + 5) dy dx$.
7. Does L'Hopital's rule apply to the limit, $\lim_{x \rightarrow 0} \left(\frac{\cos x}{x} \right)$?

8. Find the rectangular co-ordinates of the point whose polar co-ordinates are given by $(r, \theta) = (4, \pi/6)$.
9. Identify the quadric surface $z = \frac{x^2}{4} + \frac{y^2}{25}$.
10. Find $r'(t)$ if $r(t) = \cos t \, i + \sin t \, j + t \, k$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. These questions carry **2** marks each.

11. Find the x-coordinates of all inflection points of $x^4 - 5x^3 + 9x^2$.
12. Let $f(x) = x^2 + px + q$. Find the values of p and q such that $f(1) = 3$ is an extreme value of f on $[0, 2]$. Is this value a maximum or minimum?
13. Find the length of the curve $y = 2x$ from $(1, 2)$ and $(2, 4)$.
14. Find the total area between the curve $y = 1 - x^2$ and the x axis over the interval $[0, 2]$.
15. Find area of the surface that is generated by revolving the portion of the curve $y^2 = x$ from origin to the point where $x = 2$ about the x-axis.
16. Evaluate : $\int_{-1}^2 \int_0^2 \int_0^1 (x^2 + y^2 + z^2) \, dx \, dy \, dz$.
17. Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to the extremity of the latus rectum.
18. Find the area of the surface that is generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$.
19. Prove that the curve $r(t) = \left(2t, \frac{2}{1+t^2} \right)$ is also the graph of the function $y = \frac{8}{4+x^2}$.

20. Describe the graph of $r(t) = (1 + 2t, -1 + 3t)$.

21. If $f(x, y) = xy$, find $D_u f(1, 2)$ for the unit vector $u = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$.

22. Find the natural domain of the vector valued function $r(t) = \cos \pi t \, i - \ln t \, j + \sqrt{t-4} \, k$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. These questions carry **4** marks each.

23. Verify Rolle's theorem for the function $f(x) = x^3 - 9x$.
24. Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$.
25. Find by double integration, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.
26. Find the length of the curve $27y^2 = 4x^3$ from $(0, 0)$ to $(3, 2)$.
27. Find the entire length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.
28. Use the Maclaurin series for $\frac{1}{1+x^2}$ to find the Maclaurin series for $\tan^{-1} x$.
29. Evaluate the integral $\int_0^{2\sqrt{4-x^2}} \int_0^x (x^2 + y^2) \, dy \, dx$ by converting to polar coordinates.
30. If $r_1(t) = 5t \, i + t \, j - t^4 \, k$ and $r_2(t) = \sin t \, i - \cos t \, j$. Find $\frac{d}{dt}(r_1 \cdot r_2)$ and $\frac{d}{dt}(r_1 \times r_2)$.
31. Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at the point $(1, -2, 0)$ in the direction of the vector $a = 2i + j - 2k$.

(6 × 4 = 24 Marks)