



Reg. No. : .....

Name : .....

First Semester B.Sc. Degree Examination, February 2018  
First Degree Programme Under CBCSS  
**COMPLEMENTARY COURSE I FOR CHEMISTRY AND POLYMER  
CHEMISTRY**

**MM 1131.2 : Mathematics I – Differentiation and Matrices  
(2014 Admission Onwards)**

Time : 3 Hours

Max. Marks : 80

**SECTION – I**

Answer all questions. Each question has 1 mark.

1. Let  $f(x) = x^2 + 3$  and  $g(x) = \sqrt{x}$ . Find  $(g \circ f)(x)$ .
2. Find  $\lim_{x \rightarrow 1} (x^7 - 2x^5 + 1)^{2017}$ .
3. Suppose that  $s = f(t) = 1 + 5t - 2t^2$  is the position function of a particle, where  $s$  is in meters and  $t$  is in seconds. Find the average velocity of the particle over the time interval  $[2, 3]$ .
4. Find the slope of the tangent line to  $y = 2x^3 + 1$  at  $x = -1$ .
5. Find the 6<sup>th</sup> Maclaurin polynomial for  $\cos x$ .
6. Describe the surface  $4x^2 + 4y^2 + z^2 + 8y - 4z = -4$ .
7. Find the critical points of the surface  $z = x^2 + y^2$ .
8. Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$ .
9. Find all solutions of  $x_1 + x_2 - x_3 - 5x_4 = 1$ .
10. Write a  $3 \times 3$  matrix having eigen values  $-1, 0, 1$ .



## SECTION - II

Answer **any eight** questions. **Each** question has **2** marks.

11. Determine whether the function  $f(x) = \begin{cases} x^2 - 4, & x \neq 2 \\ 4 & x = 2 \end{cases}$  is continuous at  $x = 2$ .
12. A robot moves in the positive direction along a straight line so that after  $t$  minutes its distance is  $s = 6t^4$  feet from the origin.
- Find the average velocity of the robot over the interval  $[2, 4]$ .
  - Find the instantaneous velocity at  $t = 2$ .
13. Given that  $f(-2) = 3$  and  $f'(-2) = -4$ , find an equation for the tangent line to the graph of  $y = f(x)$  at  $x = -2$ .
14. Verify Rolle's theorem for the function  $f(x) = \cos x$  in the interval  $[\pi/2, 3\pi/2]$ .
15. Find the interval of convergence and radius of convergence of the power series  $\sum_{k=0}^{\infty} k! x^k$ .
16. Approximate  $\cos 0.1$  to five decimal-place accuracy using the Maclaurin series for  $\cos x$ .
17. Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  where  $z = \cos(x^5 y^4)$ .
18. Let  $f$  be a differentiable function of one variable and let  $z = f(x^2 + y^2)$ .  
Find  $y \frac{\partial z}{\partial x} - x \frac{\partial z}{\partial y}$ .
19. Find the inverse of  $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$  by performing elementary transformations.
20. Determine whether the following system of equations is consistent.  
 $3x + 4y = 5$   
 $6x + 8y = 10$



21. Find the eigen values of the matrix  $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ .
22. Determine whether the matrix  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  is diagonalizable.

## SECTION - III

Answer any six questions. Each question has 4 marks.

23. Let  $s = \frac{100}{t^2 + 12}$  be the position function of a particle moving along a coordinate line, where  $s$  is in feet and  $t$  is in seconds. Find the maximum speed of the particle for  $t \geq 0$ , and find the direction of motion of the particle when it has its maximum speed.
24. Find the absolute extrema, if any, of the function  $f(x) = e^{(x^2 - 3x^2)}$  on the interval  $(0, +\infty)$ .
25. Verify that the hypotheses of the mean-value theorem are satisfied for the function  $f(x) = x^3 + x - 4$  in the interval  $[-1, 2]$  and find all values of  $c$  in the interval that satisfy the conclusion of the theorem.
26. Find an interval containing  $x = 0$  throughout which  $f(x) = \cos x$  can be approximated by  $p(x) = 1 - (x^2/2!)$  to three decimal place accuracy.
27. Use appropriate forms of the chain rule to find  $\frac{\partial z}{\partial u}$  and  $\frac{\partial z}{\partial v}$  where  $z = x/y$ ;  $x = 2 \cos u$ ,  $y = 3 \sin v$ .
28. Locate all relative extrema and saddle points of  $f(x, y) = 4xy - x^4 - y^4$ .
29. Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$  by reducing it into Echelon form.
30. Find all solutions of the following system of linear equations.  
 $x_1 - 2x_2 + x_3 - x_4 = 0$   
 $x_1 + x_2 - 2x_3 + 3x_4 = 0$   
 $4x_1 + x_2 - 5x_3 + 8x_4 = 0$   
 $5x_1 - 7x_2 + 2x_3 - x_4 = 0$
31. Find the eigen values and eigen vectors of the matrix  $\begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$ .



## SECTION - IV

Answer **any two** questions. **Each** question has **15** marks.

32. a) Find all values of  $x$  for which the line that is tangent to  $y = 3x - \tan x$  is parallel to the line  $y - x = 2$ .
- b) A fertilizer producer finds that it can sell its product at a price of  $p = 300 - 0.1x$  dollars per unit when it produces  $x$  units of fertilizer. The total production cost (in dollars) for  $x$  units is  $C(x) = 15,000 + 125x + 0.025x^2$ . If the production capacity of the firm is at most 1000 units of fertilizer in a specified time, how many units must be manufactured and sold in that time to maximize the profit?

33. a) The function  $s(t) = t^4 - 4t^2 + 4$ ;  $t \geq 0$  describes the position of a particle moving along a coordinate line, where  $s$  is in feet and  $t$  is in seconds.
- Find the velocity and acceleration functions.
  - Find the position, velocity, speed and acceleration at time  $t = 1$ .
  - At what times is the particle stopped?
  - When is the particle speeding up? Slowing down?
  - Find the total distance traveled by the particle from time  $t = 0$  to time  $t = 5$ .
- b) Find the first three nonzero terms in the Maclaurin series for  $\tan x$ .

34. a) Solve for  $x$  and  $y$  in terms of  $u$  and  $v$  and then find the Jacobian  $\frac{\partial(x, y)}{\partial(u, v)}$ ;  
 $u = e^x, v = ye^{-x}$ .
- b) Using Lagrange multipliers find the point on the line  $y = 2x + 3$  that is closest to  $(4, 2)$ .

35. a) Show that the system

$$2x_1 - x_2 + 3x_3 = 8$$

$$-x_1 + 2x_2 + x_3 = 4$$

$$3x_1 + x_2 - 4x_3 = 0$$

is consistent and solve it.

- b) Show that the matrix  $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$  is diagonalizable. Also find the diagonal form and a diagonalizing matrix.