



Reg. No. :

Name :

First Semester B.Sc. Degree Examination, February 2018**First Degree Programme Under CBCSS****Complementary Course I for Chemistry and Polymer Chemistry****MM 1131.2 : MATHEMATICS I – DIFFERENTIATION AND MATRICES****(2013 Admission)**

Time : 3 Hours

Max. Marks : 80

SECTION – IAnswer **all** questions. **Each** question has **1** mark.

1. Let $f(x) = x + 5$ and $g(x) = x^2 - 3$. Find $(g \circ f)(0)$.

2. Find $\lim_{x \rightarrow 2} (-x^2 + 5x - 2)$.

3. Suppose that $s = f(t) = 1 + 5t - t^2$ is the position function of a particle, where s is in meters and t is in seconds. Find the average velocity of the particle over the time interval $[1, 2]$.4. Find the slope of the tangent line to $y = 2x^3$ at $x = 1$.5. Find the 4th Maclaurin polynomial for $\cos x$.6. Describe the surface $x^2 + y^2 + z^2 + 4y - 2z = -1$.7. Find the critical points of the surface $z = x^3 + y^3$.

8. Find the rank of the matrix
$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

9. Find h and k such that the system
$$\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k \end{aligned}$$
 has no solution.10. Write a 3×3 matrix having eigen values 2, 3, 5.



SECTION - II

Answer **any eight** questions. Each question has **2** marks.

11. For what value of a the function $f(x) = \begin{cases} x; & x < -2 \\ ax^2; & x \geq -2 \end{cases}$ is continuous at every x .

12. A body moves such that $s = t^2 - 3t + 2$; $0 \leq t \leq 2$ where s is in meters and t is in seconds.

- Find the displacement of the body
- Find the instantaneous velocity at $t = 1$.

13. Find an equation for the line perpendicular to the tangent to the curve $y = x^3 - 4x + 1$ at the point $(2, 1)$.

14. Verify Rolle's theorem for the function $f(x) = \sin x$ in the interval $[0, \pi]$.

15. Find the interval of convergence and radius of convergence of the power series $\sum_{k=0}^{\infty} \frac{(x-2)^k}{10^k}$.

16. Approximate $\cos 3^\circ$ to four decimal-place accuracy using the Maclaurin series for $\cos x$.

17. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(4, -5)$ where $z = x^2 + 3xy + y^2 - 1$.

18. Let $f(x, y) = y^2 e^x + y$. Find f_{xyy} .

19. Find the rank of $\begin{bmatrix} 1 & -2 & -1 \\ 0 & 2 & 2 \\ -2 & 2 & 0 \end{bmatrix}$.

20. Determine whether the following system of equations is consistent.

$$\begin{aligned} x_1 - 2x_2 - x_3 + 3x_4 &= 0 \\ -2x_1 + 4x_2 + 5x_3 - 5x_4 &= 3 \\ 3x_1 - 6x_2 - 6x_3 + 8x_4 &= 2. \end{aligned}$$

21. Find the eigen values of the matrix $\begin{bmatrix} 3 & -2 \\ 3 & 4 \end{bmatrix}$.

22. Determine whether the matrix $\begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}$ is diagonalizable.



SECTION - III

Answer **any six** questions. Each question has **4** marks.

23. Find the absolute extrema of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$ and determine where these values occur.
24. A position function of a particle moving along a coordinate line is given by $s = 5t^2 - 20t$. Analyse the motion of the particle for $t \geq 0$.
25. Verify that the hypotheses of the Mean-Value Theorem are satisfied for the function $f(x) = x - \frac{1}{x}$ in the interval $[3, 4]$ and find all values of c in the interval that satisfy the conclusion of the theorem.
26. Suppose that the function f is represented by the power series $f(x) = 1 - \frac{x-5}{3} + \frac{(x-5)^2}{3^2} - \frac{(x-5)^3}{3^3} + \dots$. Find the domain of f and give $f(3)$ and $f(6)$.
27. Use an appropriate form of the chain rule to find dw/dt . $w = 5 \cos xy - \sin xz$; $x = 1/t$, $y = t$, $z = t^3$.
28. Locate all relative extrema and saddle points of $f(x, y) = x^2 + xy - 2y - 2x + 1$.
29. Find the rank of the matrix $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing it into Echelon form.
30. Find all solutions of the following system of linear equations :
- $$\begin{aligned} 2x_1 - 2x_2 + 5x_3 + 3x_4 &= 0 \\ 4x_1 - x_2 + x_3 + x_4 &= 0 \\ 3x_1 - 2x_2 + 3x_3 + 4x_4 &= 0 \\ x_1 - 3x_2 + 7x_3 + 6x_4 &= 0. \end{aligned}$$

31. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.



SECTION - IV

Answer **any two** questions. **Each** question has **15** marks.

32. a) Given that the tangent line to $y = f(x)$ at the point $(1, 2)$ passes through the point $(-1, -1)$, find $f'(1)$.

b) Evaluate $\lim_{x \rightarrow 0} (1 + \sin x)^{1/x}$.

33. a) The function $s(t) = t^4 - 4t^2 + 4$; $t \geq 0$ describes the position of a particle moving along a coordinate line, where s is in feet and t is in seconds.

i) Find the velocity and acceleration functions.

ii) Find the position, velocity, speed and acceleration at time $t = 1$.

iii) At what times is the particle stopped.

iv) When is the particle speeding up ? Slowing down ?

v) Find the total distance traveled by the particle from time $t = 0$ to time $t = 5$.

b) Find the first three nonzero terms in the Maclaurin series for $\tan x$.

34. a) Solve for x and y in terms of u and v and then find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$:

$$u = e^x, v = ye^{-x}.$$

b) Using Lagrange multipliers find the point on the line $2x - 4y = 3$ that is closest to the origin.

35. a) Show that the system

$$x_1 + x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 3x_3 = 14$$

$$x_1 + 4x_2 + 7x_3 = 30$$

is consistent and solve it.

b) Show that the matrix $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable. Also find the diagonal

form and a diagonalizing matrix.
