



Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, July 2017

First Degree Programme under CBCSS

MATHEMATICS

Foundation Course – II

MM 1221 : Foundations of Mathematics

(2014 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are compulsory. They carry 1 mark each :

1. Find the least non-negative solution of $12x \equiv 5 \pmod{25}$.
2. Give a complete set of representatives for $\mathbb{Z}/9\mathbb{Z}$.
3. In which congruence class in $\mathbb{Z}/7\mathbb{Z}$ is 2016 ?
4. Define Euler's phi function.
5. Find the relative extrema of $f(x) = 1 + 8x - 3x^2$.
6. Evaluate $\int (x^3 + x + 5) dx$.
7. If $\int_0^1 f(x) dx = \frac{1}{2}$, $\int_1^2 f(x) dx = \frac{1}{4}$ evaluate $\int_0^2 f(x) dx$.
8. Evaluate $\int \frac{1}{1+3x^2} dx$.
9. Find the polar coordinates of the point whose rectangular coordinates $(0, -2)$ satisfies $r \geq 0$ and $0 \leq \theta \leq 2\pi$.
10. Express the Cartesian equation $9xy = 4$ in polar coordinates.



SECTION - II

Answer **any 8** questions from among the questions **11 to 22**. These questions carry **2 marks each** :

11. What are the units of $\mathbb{Z}/5\mathbb{Z}$? Also find the elements that are not units.
12. Find the inverses of 2, 4, 5, 7 in $\mathbb{Z}/13\mathbb{Z}$.
13. Solve $[3]X = [11]$ in $\mathbb{Z}/17\mathbb{Z}$.
14. Verify that $\phi(ab) = \phi(a)\phi(b)$ for $a = 7$, $b = 8$.
15. Write down the addition and multiplication table for arithmetic modulo 3.
16. If n is any number, prove that $n(n+1)(n+2)$ is divisible by 6.
17. Use a double integral to find the volume under the plane $z = 2x + y$ over the rectangle $R = \{(x, y) : 3 \leq x \leq 6, 1 \leq y \leq 2\}$.
18. State the fundamental theorem of calculus and obtain $\int_{-\pi/2}^{\pi/2} \sin \theta \, d\theta$.
19. A particle moves with velocity $v(t) = \sin t$, $0 \leq t \leq \frac{\pi}{2}$ (miles/second) along an s -axis. Find the displacement and distance travelled by the particle during the given time interval $0 \leq t \leq \frac{\pi}{2}$.
20. Evaluate $\int \cos 4\theta \sqrt{2 - \sin 4\theta} \, d\theta$.
21. Sketch the region whose signed area is represented by the definite integral $\int_{-1}^4 x \, dx$ and evaluate the integral using an appropriate formula from geometry.
22. Find the area of the region enclosed by the rose curve $r = \cos 2\theta$.



SECTION - III

Answer any 6 questions from among the questions 23 to 31. These questions carry 4 marks each :

23. Define the order of a modulo m and find the order of the non-zero elements of $\mathbb{Z}/7\mathbb{Z}$.
24. If e is the order of a modulo m and $a^e \equiv 1 \pmod{m}$ then show that e divides ϕ .
25. Prove that $[a]$ is a unit of $\mathbb{Z}/m\mathbb{Z}$ if and only if $(a, m) = 1$.
26. Given the position function and velocity function for a particle in free fall motion as $s(t) = s_0 + v_0 t - \frac{1}{2}gt^2$ and $v(t) = v_0 - gt$, a projectile fired vertically upward from ground level with an initial velocity of 16ft/s :
- a) How long will it take for the projectile to hit the ground ?
- b) How long will the projectile be moving upward ?
27. Find the area of the region enclosed by the parabola $y = 2x - x^2$ and the x -axis.
28. Define the average value of a function f on $[a, b]$. Suppose that the acceleration function of a particle moving along a coordinate line is $a(t) = t + 1$. Find the average acceleration of the particle over the time interval $0 \leq t \leq 5$ by integrating.
29. Find the volume of the solid generated when the region between the graph of the equation $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x -axis.
30. Show that the lateral surface area S of a right circular cone with height h and base radius r is $S = \pi r \sqrt{r^2 + h^2}$.
31. Find a number in the closed interval $\left[\frac{1}{2}, \frac{3}{2}\right]$ such that the sum of the number and its reciprocal is :
- a) as small as possible
- b) as large as possible.



SECTION - IV

Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each** :

32. a) State and prove Fermat's theorem.
b) Show that if 7 does not divide n , then 7 divides $n^{12} - 1$.
33. a) i) State and prove Euler's theorem.
ii) Show that $3^{2n} + 24n - 1$ is divisible by 32 for any integer n .
b) Find some b so that $\{0, b, b^2, \dots, b^{m-1}\}$ is a complete set of representatives for $\mathbb{Z}/m\mathbb{Z}$ when $m = 5$.
34. a) Use cylindrical shells to find the volume of the solid that is generated when the region under the curve $y = x^3 - 3x^2 + 2x$ over $[0, 1]$ is revolved about the y -axis.
b) Find the arc length of the curve $y = x^{\frac{2}{3}}$ from $(1, 1)$ to $(2, 2\sqrt{2})$.
35. a) State focus directrix property of conics and derive polar equations of the conic sections from their focus-directrix characterization.
b) Sketch the graph of $r = \frac{1}{1 + \cos \theta}$.
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