

Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

MM 1661.1 : Graph Theory (Elective)

(2018 Admission Regular)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define a simple graph.
2. How many edges do K_{20} have?
3. Define an empty graph.
4. Define a connected graph.
5. A tree with n vertices has ... edges.
6. Define a cut vertex of a graph.
7. Define a Hamiltonian graph.
8. Is K_4 Eulerian?
9. How many regular polyhedra are there?
10. State Euler's formula in a connected plane graph.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Define a bipartite graph with example.
12. Define a regular graph. Draw a 2-regular graph.
13. Let G be a graph in which there is no pair of adjacent edges. What can you say about the degrees of the vertices in G ?
14. Define complement of a graph. Give an example.
15. Define join of two graphs.
16. Define adjacency matrix of a graph G . Give an example.
17. Draw all non-isomorphic trees with 4 vertices.
18. Define vertex connectivity of a graph with example.
19. Define a maximal non-Hamiltonian graph. Give example.
20. Define an Euler tour. Give an example.
21. Define a Jordan curve.
22. State Cayley's theorem on spanning trees.
23. Define a plane graph. Give an example.
24. Define a polyhedral graph.
25. State Kuratowski's theorem on planar graphs.
26. How can we obtain a subdivision of a graph G .

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4 marks** each.

27. Prove that in any graph there is an even number of odd vertices.
28. Given two vertices u and v of a graph G . Prove that every u - v walk contains a u - v path.
29. Define the following in a connected graph
 - (a) Distance between two vertices
 - (b) Eccentricity of a vertex
 - (c) Radius of a graph
 - (d) Diameter of a graph.
30. Let u and v be distinct vertices of a tree T . Prove that there is precisely one path from u to v .
31. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G .
32. Let G be a graph with n vertices where $n \geq 2$. Prove that G has at least two vertices which are not cut vertices.
33. Describe Konigsberg bridge problem.
34. Define closure of a graph. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.
35. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.
36. Prove that the complete graph K_5 is non planar.

37. If G is a simple planar graph then prove that G has a vertex v of degree less than 6.
38. Let G be a simple graph with at least 11 vertices. Prove that either G or its complement $\overline{G}$ must be non-planar.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15 marks** each.

39. Let G be a non-empty graph with at least two vertices. Prove that G is bipartite if and only if it has no odd cycles.
40. Define a spanning tree with example. Prove that a graph G is connected if and only if it has a spanning tree.
41. Let G be a simple graph with at least three vertices. Then prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G there are two internally disjoint u - v paths in G .
42. Prove that a connected graph is Euler if and only if the degree of every vertex is even.
43. (a) Describe travelling salesman problem.
 (b) If G is a simple graph with n vertices where $n \geq 3$ and the degree $d(v) \geq n/2$ for every vertex v of G then prove that G is Hamiltonian.
44. Let P be a convex polyhedron and G be its corresponding polyhedral graph. For each $n \geq 3$ let v_n denote the number of vertices of G of degree n and let f_n denote the number of faces of G of degree n . Prove that.
 (a) $\sum_{n \geq 3} n v_n = \sum_{n \geq 3} n f_n = 2e$, where e is the number of edges of G .
 (b) The polyhedron P and so the graph G has at least one face bounded by a cycle of length n for either $n = 3, 4$ or 5 .

(2 × 15 = 30 Marks)