



Reg. No. :

Name :

First Semester B.Sc. Degree Examination, February 2018
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM 1141 : Methods of Mathematics
(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

(All the questions are **compulsory**. Each question carries 1 mark).

1. State the well ordering principle.
2. Find the g.c.d. of 28 and 42.
3. For all integers a, b, k and n , prove that if $a \equiv b \pmod{n}$, then $ka \equiv kb \pmod{n}$.
4. Find the smallest number ≥ 0 which is congruent to 177 (mod 9).
5. Find the amplitude and period of $y = 3 \sin 2\pi x$.
6. State intermediate value theorem.
7. Find $\frac{dy}{dx}$ if $y = -3x^{-8} + 2\sqrt{x}$.
8. Find the foci, vertices and the ends of the minor axis of the ellipse $9x^2 + y^2 = 9$.
9. Define an ellipse.
10. Define Fibonacci sequence.



SECTION - II

(Answer any 8 question. Each question carries 2 marks).

11. Find $\lim_{x \rightarrow -4} \frac{2x+8}{x^2+x-12}$.
12. Find the new co-ordinates of the point (2, 4) if the co-ordinate axes are rotated through an angle of $\theta = 30^\circ$.
13. Represent 1976 in base 2.
14. Using Euclid's algorithm finds the g.c.d. of 121 and 365.
15. Find the natural domain of $f(x) = \sqrt{x^2 - 5x + 6}$.
16. Prove that $\sqrt{2}$ is irrational.
17. Express $h(x) = (x - 4)^5$ as a composition of two functions.
18. Let $y = x^2 + 1$, find the instantaneous rate of change of y with respect to x when $x = -4$.
19. Find $y^{\left(\frac{\pi}{4}\right)}$ if $y(x) = \sec x$.
20. Let $F(x) = 5f(x) + 2g(x)$. Find $F'(2)$, given that $f(2) = -1$, $f'(2) = 4$, $g(2) = 1$ and $g'(2) = -5$.
21. If a divides bc and $(a, b) = 1$, then prove that a divides c .
22. Find $\lim_{x \rightarrow 0} \frac{\sin 6x}{\sin 8x}$.

SECTION - III

(Answer any 6 question. Each question carries 4 marks).

23. Prove that there are infinitely many primes.

24. If $m \neq n$ prove that the Fermat's numbers $F(m)$ and $F(n)$ are relatively prime.

25. i) Show that $\frac{d}{dx} \tan x = \sec^2 x$.

ii) Find $\frac{dy}{dx}$ if $y = \frac{\sin x}{1 + \cos x}$.

26. i) Find the slopes of the sides of the triangle with vertices $(0, 3)$, $(2, 0)$ and

$$\left(6, \frac{8}{3}\right).$$

ii) Is this a right triangle, justify your answer.

27. Prove that if f is differentiable at $x = x_0$, the f must be continuous at x_0 .

28. Define inflection point and find the inflection points of $f(x) = \sin x$ on $[0, 2\pi]$.

29. Let a and b be two natural numbers and suppose the remainder on dividing a by m is r and the remainder on dividing b by m is s . Then prove that $a \equiv b \pmod{m}$ if and only if $r = s$.

30. Find the vertices, foci and asymptotes of the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-4)^2}{4} = 1$.

31. Use slopes to determine whether the given points lie on the same line

a) $(1, 1)$, $(-2, -5)$ and $(0, -1)$

b) $(-2, 4)$, $(0, 2)$ and $(1, 5)$.

SECTION - IV

(Answer any 2 question. Each question carries 15 marks).

32. i) a) Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes

$$x^3 + y^3 = 3xy.$$

b) Find an equation for the tangent line to the Folium of Descartes at the point

$$\left(\frac{3}{2}, \frac{3}{2}\right).$$



- c) At what point in the first quadrant is the tangent line to the Folium of Descartes horizontal.

ii) Let $f(x) = \frac{1}{x+2}$, $x < -2$, $f(x) = x^2 - 5$, $-2 < x \leq 3$, $f(x) = \sqrt{x+13}$, $x > 3$ then

find (a) $\lim_{x \rightarrow -2} f(x)$, (b) $\lim_{x \rightarrow 0} f(x)$, (c) $\lim_{x \rightarrow 3} f(x)$.

33. i) a) Find the derivative with respect to x of $(x) = \sqrt{x}$.

b) Find the slope of the curve $y = \sqrt{x}$ at $x = 9$.

c) Find the limits of $f'(x)$ as $x \rightarrow 0^+$ and as $x \rightarrow +\infty$.

ii) Sketch the graph $x^2 + 2y^2 = 4$.

iii) Locate the relative maxima and minima of $f(x) = x^4 - 2x^2$.

34. i) Let the axes of xy - co-ordinate system have been rotated about the origin through an angle $\theta = 45^\circ$ to produce $x'y'$ - co-ordinate system. Find an equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$ - co-ordinate system.

ii) Sketch the graph $r = \frac{6}{2 + \cos \theta}$ in polar co-ordinates.

35. i) Find the parametric equations with parameter θ for a cycloid.

ii) A university parking lot charges \$3.00 per day but offers a \$40.00 monthly sticker with which the student pays only \$0.25 per day, then

a) Find equations for the cost C of parking x days per month under both payment methods and equations for $0 \leq x \leq 30$.

b) Find the value of x for which the graphs intersect and discuss the significance of this value.