

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) Find the volume of the solid generated when the region enclosed by the curves $x = \sqrt{y}$ and $x = \frac{y}{4}$ is revolved about the x-axis. **7**
- (b) Find the area of the surface that is generated by revolving the portion of the curve $x = \sqrt{9 - y^2}$ between $y = -2$ and $y = 2$ about the y-axis. **8**
33. A shell, fired from a cannon, has a muzzle speed of 800 ft/s. The barrel makes an angle of 45° with horizontal and the barrel opening is assumed to be at ground level.
- (a) Find parametric equations for the shell's trajectory relative to the coordinate system
- (b) How high does the shell rise?
- (c) How far does the shell travel horizontally?
- (d) What is the speed of the shell at its point of impact with the ground?
34. (a) Show that $\vec{F} = 2xy^3\hat{i} + (1 + 3x^2y^2)\hat{j}$ is conservative and find a potential function for it. **7**
- (b) Find the flux of the vector field $\vec{F}(x, y, z) = z\hat{k}$ across the outward oriented sphere $x^2 + y^2 + z^2 = a^2$. **8**
35. Verify Gauss's divergence theorem for $\vec{F}(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$ taken over the region bounded by the sphere $x^2 + y^2 + z^2 = 1$.

(2 × 15 = 30 Marks)

(Pages : 4)

R – 3114

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II — INTEGRATION AND VECTORS

(2014-2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions. Each question carries **1** mark.

1. Evaluate $\int \sec x (\sec x + \tan x) dx$.
2. Suppose that a particle moves so that its velocity at time t is $v(t) = t^3 - 3t^2 + 2t$. Find the displacement of the particle during the time interval $0 \leq t \leq 3$.
3. Find the average value of the function $f(x) = 3x$ over the interval $[1, 3]$.
4. Evaluate $\int_2^4 \int_0^1 x^2 y dx dy$.
5. Find the natural domain of $\vec{r}(t) = (\ln t - 1)\hat{i} + e^t \hat{j} + \sqrt{t}\hat{k}$.
6. Define curvature of a smooth curve.

7. Find the divergence of the vector field $\vec{F}(x, y, z) = xz^3\hat{i} + 2y^4x^2\hat{j} + 5z^2y\hat{k}$.
8. State Green's theorem.
9. Use the fundamental theorem of work integrals to evaluate $\int_{(0,0)}^{(1,1)} \nabla \phi \cdot d\vec{r}$ where $\phi(x, y) = xy$.
10. State stoke's theorem.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Find the position function of a particle that moves with velocity $v(t) = \cos \pi t$ along a coordinate line, assuming that the particle has coordinate $s = 4$ at time $t = 0$.
12. Find the work done by the force field $\vec{F}(x, y) = xy\hat{i} + x^2\hat{j}$ on a particle that moves along the parabola $x = y^2$ from $(0, 0)$ to $(1, 1)$.
13. Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.
14. Use polar coordinates to evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} dA$.
15. Evaluate $\iiint_G xy \sin(yz) dV$ where G is the rectangular box defined by the inequalities $0 \leq x \leq \pi$, $0 \leq y \leq 1$, $0 \leq z \leq \pi/6$.
16. Find a vector equation of the line tangent to the graph of $\vec{r}(t) = t^2\hat{i} - \frac{1}{t+1}\hat{j} + (4-t^2)\hat{k}$ at the point $(4, 1, 0)$.
17. A bug, starting at the reference point $(1, 0, 0)$ of the helix $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$ walks up the helix for a distance of 10 units. What are the bug's final coordinates?
18. Find the unit tangent vector to the graph of $\vec{r}(t) = \ln t\hat{i} + t\hat{j}$ where $t = e$.
19. A particle moves through the space in such a way that its velocity is $\vec{v}(t) = \hat{i} + t\hat{j} + t^2\hat{k}$. Find the coordinates of the particle at time $t = 1$ given that the particle is at the point $(-1, 2, 4)$ at time $t = 0$.

20. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the radius vector, prove that $\nabla(\|\vec{r}\|) = \frac{\vec{r}}{\|\vec{r}\|}$.
21. Evaluate the line integral $\int_C ydx + xdy$ using Green's theorem, where C is the unit circle oriented counter clockwise.
22. Use a line integral to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

23. A penny is released from rest neat the top of the Empire State Building at a point that is 1250 ft above the ground. Assuming that the free-fall model applies, how long does it take for the penny to hit the ground, and what is its speed at the time of impact.
24. Find the arc length of the curve $y = x^{3/2}$ from $(1, 1)$ to $(2, 2\sqrt{2})$.
25. Use double integral to find the volume in the first octant bounded by the coordinate planes, the plane $y = 4$ and the plane $\frac{x}{3} + \frac{z}{5} = 1$.
26. Evaluate $\int_0^1 \int_{4x}^4 e^{-y^2} dy dx$ by changing the order of integration.
27. Use cylindrical coordinates to find the volume of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$.
28. Find the curvature and radius of curvature of the curve $x = e^t \cos t$, $y = e^t \sin t$, $z = t$ at the point $t = 0$.
29. Suppose that a particle moves through the space so that its position vector at time t is $\vec{r}(t) = 3 \sin t\hat{i} + 2 \cos t\hat{j} - \sin(2t)\hat{k}$. Find the scalar tangential and normal components of acceleration at time $t = \pi/2$. Also, find the vector tangential and normal components of acceleration at time $t = \pi/2$.
30. A heat-seeking particle is located at the point $(2, 3)$ on a flat metal plate whose temperature at a point (x, y) is $T(x, y) = 10 - 8x^2 - 2y^2$. Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.
31. Evaluate $\iint_{\sigma} \sqrt{x^2 + y^2 + z^2} dS$ where σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 1$.

(6 × 4 = 24 Marks)