



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2019

First Degree Programme under CBCSS

MATHEMATICS

Elective

MM 1661.1 : Graph Theory

(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Define a simple graph.
2. The number of odd vertices in a graph is always _____
3. What is a spanning subgraph ?
4. Define outdegree.
5. Is the following graph connected ?



6. Define Euler graph.
7. What is a unicursal graph ?
8. Define radius of a graph.
9. A tree with n vertices has _____ edges.
10. What is maximal tree of a graph ?

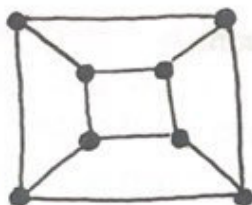




SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Write any two applications of graph theory with suitable explanation.
12. Write the adjacency matrix of C_4 .
13. Prove that the sum of degrees is equal to twice the number of edges.
14. Label the following graphs to prove that they are isomorphic.



15. Prove that a graph G is disconnected if and only if the vertex set can be partitioned into 2 non-empty disjoint subsets V_1 and V_2 such that there is no edge having one end vertex in V_1 and another in V_2 .
16. Is the following graph Euler graph ? Explain.



17. Explain Chinese Postman problem.
18. State a characterization theorem for Euler digraph. Illustrate with an example.
19. Prove that there is one and only one path between every pair of vertices in a tree T .
20. Prove that a graph with n vertices, $n - 1$ edges and no circuits is connected.



21. Prove that a graph G is a tree if and only if it is minimally connected.

22. Define spanning tree. Find a spanning tree of the following graph.



SECTION – III

Answer **any 6** questions from among the questions **23 to 31**. These questions carry **4 marks each**.

23. Draw all non-isomorphic graphs on 4 vertices. How many of them are self-complementary? How many are connected?

24. Define spanning subgraph and induced subgraph. Is P_4 a spanning subgraph of K_4 ? Is it an induced subgraph? Explain.

25. Define incidence matrix. Draw the graph with incidence matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

26. Prove that a graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ components.

27. In a connected graph G with exactly $2k$ odd vertices, prove that there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

28. Prove that every tree has either one or two centers.

29. Prove that every connected graph has at least one spanning tree.

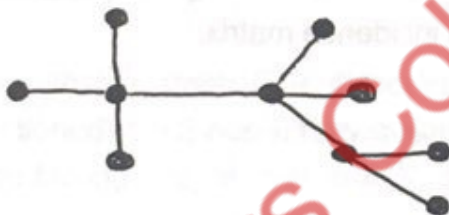


30. Draw planar representations of K_4 and a cube.
31. Prove that in any simple connected planar graph with f regions, n vertices and e edges, $e \geq \frac{3f}{2}$ and $e \leq 3n - 6$.

SECTION - IV

Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each**.

32. Explain in detail multicolour cube puzzle and its graph theoretic model.
33. Explain decanting problem with its graph theoretic formation.
34. a) Prove that, if in a graph G there is one and only one path between every pair of vertices, then G is a tree.
b) Prove that a tree with n vertices had $n - 1$ edges.
c) Find the center of the following tree.



35. Define planar graphs. State a necessary and sufficient condition for a graph G to be planar. Explain Four Colour Theorem and its graph theoretic interpretation.