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G – 1391

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2019

First Degree Programme Under CBCSS

Mathematics

Core Course – XII

MM 1644 : GRAPH THEORY

(2013 Admn.)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are **compulsory**. They carry 1 mark each.

1. Draw a simple graph with 5 vertices and 7 edges.
2. Define a regular graph.
3. Define a null graph.
4. How many edges are there for a complete graph with  $n$  vertices ?
5. Define a symmetric digraph.
6. Define components of a graph.
7. Draw a unicursal graph.
8. Draw  $K_4$  and find two edge disjoint subgraphs of  $K_4$ .
9. Define a tree.
10. Draw a bipartite graph which is not a tree.

P.T.O.



## SECTION - II

Answer **any 8** questions from among the questions **11 to 22**. These questions carry **2 marks each**.

11. Prove that  $\sum_{\text{odd}} d(V_K) = \text{an even number}$ , where  $V_K$ 's are odd degree vertices in a graph.
12. Draw a connected graph that becomes disconnected when any edge is removed from it.
13. Prove that a graph  $G$  is disconnected iff its vertex set  $V$  can be partitioned into two non-empty disjoint sets  $V_1$  and  $V_2$  such that there exists no edge in  $G$  whose one end vertex is in subset  $V_1$  and the other in subset  $V_2$ .
14. Draw two isomorphic digraphs.
15. Define a complete symmetric digraph.
16. State a necessary and sufficient condition for a digraph to be Euler.
17. State the Teleprinter's problem.
18. Draw all trees of  $n$  labelled vertices for  $n = 1, 2, 3$ .
19. Draw a plane representation of the complete graph on 4 vertices.
20. Show that in any simple, connected planar graph with  $f$  regions,  $n$  vertices and  $e$  edges,  $e \geq \frac{3}{2}f$ , where  $e > 2$ .
21. State the Kuratowski's theorem.
22. Show that a minimally connected graph is a tree.

## SECTION - III

Answer **any 6** questions from among the questions **23 to 31**. These questions carry **4 marks each**.

23. Write a short note on the origin of graph theory.
24. Describe finite and infinite graphs. Illustrate with examples.



25. How graphs can be described using matrices ? Give examples.
26. Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.
27. Prove that a connected graph is unicursal iff it has exactly two vertices of odd degree.
28. Prove that there is one and only one path between every pair of vertices in a tree  $T$ .
29. Prove that every tree has either one or two centers.
30. Prove that a graph can be embedded in the surface of a sphere iff it can be embedded in a plane.
31. Describe the graph theoretic version of the four colour problem.

#### SECTION – IV

Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each**.

32. Prove that a tree with  $n$  vertices has  $n - 1$  edges.
  33. Prove that a simple graph with  $n$  vertices and  $k$  components have at most  $(n - k)(n - k + 1)/2$  edges.
  34. Prove that a connected graph  $G$  is an Euler graph iff all vertices of  $G$  are of even degree.
  35. Prove that a connected planar graph with  $n$  vertices and  $e$  edges has  $e - n + 2$  regions.
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