



Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, December 2017
First Degree Programme under CBCSS
Complementary Course for Chemistry & Polymer Chemistry
MM 1331.2 : MATHEMATICS – III (Theory of Equations and Vector
Analysis)
(2013 Admissions)

Time : 3 Hours

Max. Marks : 80

All the first 10 questions are compulsory. Each question carries 1 mark.

1. Find the equation with rational coefficient whose roots are $1 + 5\sqrt{-1}$ and $5 - \sqrt{-1}$.
2. If $\alpha_1, \alpha_2, \dots, \alpha_n$ are the roots of the equation $a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n = 0$ then $\alpha_1 + \alpha_2 + \dots + \alpha_n =$ _____
3. If $\alpha_1, \alpha_2, \dots, \alpha_n$ are the roots of the equation $x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n = 0$, then give the roots of the equation $a_0 + a_1x + \dots + a_{n-1}x^{n-1} + a_nx^n = 0$.
4. Define reciprocal equation.
5. Give the natural domain for the vector valued function

$$r(t) = 6ti + \frac{1}{t}j + \frac{3}{t^3}k.$$

6. Find the volume of the tetrahedron with co-terminal edges representing the vectors $i + j$, $i - j$ and $2k$.
7. Find the parametric equations that correspond to the equation $r = 3t^2i - 2j$.
8. Find the gradient of $f(x, y) = x^2 + \frac{y^2}{2}$.



9. Give the value of $\text{div}(\hat{r})$, where $\hat{r}$ is the unit vector in the direction of r .
10. Verify that the scalar function $\phi = x^2 + y^2$ satisfies the Laplace equation
 $\nabla^2 \phi = 0$. (10x1=10 Marks)

Answer **any 8** questions from among the questions **11 to 22**. These questions carry **2 marks each**.

11. Transform the equation $x^3 - 6x^2 + 10x - 3 = 0$ into an equation lacking the second term.
12. Find the equation whose roots are the squares of the roots of $x^4 + x^3 + 2x^2 + x + 1 = 0$.
13. Show that $7x^5 - 100x^2 + 22 = 0$ has at least two imaginary roots.
14. Find the condition that the roots of $ax^2 + bx + c = 0$ may be both positive.
15. Find the unit tangent vector at a point t to the curve

$$r = a \cos t \mathbf{i} + a \sin t \mathbf{j}.$$

16. Find the directional derivative of

$$f(x, y) = x^2 + xy \text{ at } P_0(1, 2) \text{ in the direction of the unit vector } u = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}.$$

17. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin and the point $(1, 1, 1)$.
18. Find the work done by the conservative field

$$F = xz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} = \nabla(xyz)$$

along any smooth curve C joining the point $(-1, 3, 9)$ to $(1, 6, -4)$.

19. Prove that $\text{curl}(\text{curl } F) = \text{grad div } F - \nabla^2 F$.
20. Find the work done by the force field $F(x, y) = xy\mathbf{i} + x^2\mathbf{j}$ on a particle that moves along the curve $C: x = y^2$ from $(0, 0)$ to $(1, 1)$.



21. Evaluate the surface integral $\iint_{\sigma} xz \, dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant.
22. Use the Divergence Theorem to find the outward flux of the vector field $F(x, y, z) = x^3 \mathbf{i} + y^3 \mathbf{j} + z^2 \mathbf{k}$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$. (8x2=16 Marks)

Answer **any 6** questions from the questions **23** to **31**. These questions carry **4 marks each**.

23. Form the polynomial equation of fourth degree with rational coefficients, one of whose roots is $\sqrt{2} + \sqrt{-3}$.
24. If α, β, γ are the roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.
25. Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetic progression if $2p^3 - 9pq + 27r = 0$.
26. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $2\alpha + 3, 2\beta + 3, 2\gamma + 3$.
27. Show that the derivative of a vector-valued function $\mathbf{r}(t)$ of constant magnitude is either the zero vector or is perpendicular to $\mathbf{r}(t)$.
28. If $\mathbf{F}$ is a vector having fixed direction, show that $\text{curl } \mathbf{F}$ is perpendicular to $\mathbf{F}$.
29. Find the length of the indicated portion of the curve $\mathbf{r}(t) = (2 \cos t) \mathbf{i} + (2 \sin t) \mathbf{j} + \sqrt{5}t \mathbf{k}, 0 \leq t \leq \pi$.
30. Using the vector equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ find the curvature of the ellipse at the end points of the major and minor axes. Deduce the curvature of the circle.
31. Verify Green's theorem in the plane for $\int_C (xy \, dx + x^2 \, dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$. (6x4=24 Marks)



Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15 marks each**.

32. If α, β are the roots of $lx^2 + mx + n = 0$, find the equation whose roots are

$$\frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha}.$$

33. Using Cardan's method, solve

$$x^3 - 6x^2 + 3x - 2 = 0$$

34. The temperature of points in space is given by $f(x, y, z) = x^2 + y^2 - z$. A mosquito located at $(1, 1, 2)$ desires to fly in such a direction that it will get cool as soon as possible. In what direction should it move?

35. Verify Stokes' theorem for $F = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$, where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(2x15=30 Marks)