

Reg. No. : .....

Name : .....

Fourth Semester B.Sc. Degree Examination, May 2021

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1431.1 – STATISTICAL INFERENCE

(2018 Admission)

Time : 3 Hours

Max. Marks : 80

Instructions: Use of statistical table and scientific calculator are permitted

## SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Define an unbiased estimator.
2. State the invariance property of consistent estimator.
3. Write the MLE of the parameter  $\theta$  based on a random sample taken from a population with pdf  $f(x) = \frac{1}{\theta}$ ,  $0 \leq x \leq \theta$ .
4. Write down two properties of moment estimators.
5. What is confidence coefficient?

6. What is power of a test?
7. Define test statistic?
8. What is P value?
9. Write down test statistic for testing the mean of a population for a large sample when population SD  $\sigma$  is unknown.
10. What is local control?

(10 × 1 = 10 Marks)

### SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. If  $f(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}$ ,  $\lambda > 0$  for  $x = 0, 1, 2, \dots$ , then estimate  $\lambda$  by the method of moments.
12. For the random sample  $x_1, x_2, \dots, x_n$  taken from Poisson with parameter  $\lambda$ , obtain an unbiased estimator of  $e^{-\lambda}$ .
13. Define consistency. Write down the consistent estimator for  $\mu$ , for a sample taken from  $N(\mu, \sigma^2)$ .
14. Write down the  $(1-\alpha)\%$  confidence interval for the proportion of a binomial population.
15. Let  $x_1, x_2, \dots, x_n$  be the random sample From a population with p.d.f  $f(x, \theta) = \theta x^{\theta-1}$ ,  $0 < x < 1$ ,  $\theta > 0$ . Find the sufficient estimator of  $\theta$ .

16. What you mean by null hypothesis and alternative hypothesis?
17. What you mean by one tailed and two tailed tests?
18. Define most powerful test
19. Give the test statistic in the case of small sample test to test the equality of means of two normal populations (a) when population SDs are known (b) when population SDs are unknown.
20. If  $X$  follow a distribution with p.d.f  $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x \geq 0, \theta > 0$ . To test  $H_0: \theta = 5$  against  $H_1: \theta = 10$ .  $H_0$  is rejected if  $X \geq 15$ . Obtain the probability of type 1 error.
21. What is method of moments?
22. What is total sum of squares and how it is decomposed?

(8 × 2 = 16 Marks)

### SECTION – C

Answer **any six** questions. Each question carries **4** marks

23. For the random sample  $x_1, x_2, \dots, x_n$  taken from  $N(\mu, \sigma^2)$ , show that the sample variance is a biased estimator of the population variance.
24. Obtain the M.L.E. of  $\lambda$ , based on random samples taken from Poisson population with parameter  $\lambda$ .
25. A random sample of size 15 from a normal population given  $\bar{x} = 3.2$  and  $s^2 = 4.24$ . Determine the 90% confidence limits for  $\sigma^2$ .

26. If  $x_1, x_2, \dots, x_n$  are random observation on a Bernoulli variate  $x$  taking the value 1 with probability  $p$  and the value 0 with probability  $(1-p)$ , show that  $\frac{\sum x_i}{n} \left(1 - \frac{\sum x_i}{n}\right)$  is a consistent estimator of  $p(1-p)$ .
27. Give the procedure of testing of hypothesis.
28. In a die throwing experiment, getting 4 or 6 is considered as a success. Suppose 9000 times the die was thrown and resulted in 3240 successes, Is the data support the claim that the die is unbiased?
29. The travel time in minutes of 5 individuals to reach their office in peak hours of a day in a city is noted before and after implementing a new traffic plan. Test whether the new plan helps to save the travel time at 5% significance level.
- | Person                                                  | A  | B  | C  | D  | E  |
|---------------------------------------------------------|----|----|----|----|----|
| Travel time in minutes Before implementing traffic plan | 65 | 35 | 80 | 40 | 50 |
| After implementing traffic plan                         | 45 | 15 | 61 | 31 | 20 |
30. Rats were grown under two experimental conditions. Two random samples of 11 and 9 rats show the sample standard deviations of their weights as 0.8 and 0.5 respectively. Assuming the weight distributions is normal, test the hypothesis that the true variances are equal at 10% significance level.
31. Explain a linear model.

(6 × 4 = 24 Marks)

## SECTION – D

Answer any **two**. Each question carries **15** marks.

32. For random sample  $x_1, x_2, \dots, x_n$  taken from  $B(1, p)$ . Show that if  $T = \sum_{i=1}^n X_i$

(a)  $\frac{T}{n}$  is an unbiased estimator of  $p$  (b)  $\frac{T(T-1)}{n(n-1)}$  is an unbiased estimator of  $p(1-p)$

(b)  $\frac{T(T-1)}{n(n-1)}$  is an unbiased estimator of  $p^2$ .

33. (a) Explain the large sample test of testing the mean of a population when the population standard deviation is unknown.

(b) Two different diets 'A' and 'B' were administered to two different groups of pigs.

The gains in weight by the diets are given below

Diet 'A': 25 32 30 34 24 14 32 24 30 31 35 25

Diet 'B': 44 34 22 10 47 31 40 30 32 35 18 21 35 29 22

Test whether the diets differ significantly as far as their effects on increasing weight is concerned

34. (a) Explain  $\chi^2$  test of goodness of fit.

(b) Four coins are tossed 640 times and the following results on the total number of heads obtained on each trial are noted. Can we conclude the hypothesis that the coins are unbiased is acceptable.

Number of heads : 0    1    2    3    4

Number of trails : 32   169   223   182   34

35. (a) Explain the three principles of design of experiments.

(b) Three different methods of analysis  $M_1, M_2, M_3$  are used to determine in parts per million the amount of a certain constituent in the sample. Each method is used by five analysts, and the results are given in the following table.

|         |   | Method |       |       |
|---------|---|--------|-------|-------|
|         |   | $M_1$  | $M_2$ | $M_3$ |
| Analyst | 1 | 7.5    | 7.0   | 7.1   |
|         | 2 | 7.4    | 7.2   | 6.7   |
|         | 3 | 7.3    | 7.0   | 6.9   |
|         | 4 | 7.6    | 7.2   | 6.8   |
|         | 5 | 7.4    | 7.1   | 6.9   |

Do these results indicate a significant variation either between the methods or between the analysts?

(2 × 15 = 30 Marks)

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