

Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XIII

MM 1645 : INTEGRAL TRANSFORMS

(2018 Admission onwards)

Time.: 3 Hours

Max. Marks : 80

PART – A

All the first ten questions are compulsory. They carry 1 mark each.

1. Find the Laplace transform of $f(t) = e^{-3t}$.
2. If $L[f(t)] = F(s)$, then $L\left\{\int_0^t t f(u) du\right\} = \underline{\hspace{2cm}}$
3. Find the inverse Laplace transform of $\frac{1}{s^2 + 4}$.
4. Define unit step function.
5. Write $L\{f''(t)\}$ in terms of $L(f)$, $f(0)$ and $f'(0)$.
6. Give an example of a periodic function which is neither odd nor even.

7. Define Fourier cosine transform of a function $f(t)$.
8. What is the standard form of Fourier series for an even function?
9. Find the fundamental period of the function $\cos(nx)$.
10. If $f(x)$ has period p then find the period of $f(2x)$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Find the Laplace transform of $f(t) = t \sin 2t$.
12. Find $L[e^{-3t} \sin 3t]$.
13. Evaluate $L^{-1}\left[\frac{3}{(s+2)^4}\right]$.
14. Find $L^{-1}\left(\frac{1}{(s+2)(s+3)}\right)$.
15. Solve the differential equation $y'' - y = t$, $y(0) = 1$, $y'(0) = 1$.
16. Define convolution of two functions and find the convolution of 1 and -1 .
17. Is $L[f(t) + g(t)] = L(f(t)) + L[g(t)]$? Explain.
18. Find the Fourier series of $f(x) = x$ for $0 < x < 2\pi$.
19. Find the Fourier sine series for the function $f(x) = \pi - x$ in $0 < x < \pi$.
20. Find the Fourier cosine transform of the function $f(x) = \begin{cases} k, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$.

21. Derive the Fourier transform of $f'(x)$, the derivative of $f(x)$.

22. Check whether the following functions are odd or even

(a) $\frac{x}{x^2+1}$

(b) $x^3 \tan \pi x$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

23. Find the Laplace transform of the function $f(t) = \begin{cases} t & t \geq 3 \\ 0 & t < 3 \end{cases}$.

24. Find the inverse transform of $(3s+1)/(s-1)(s^2+1)$.

25. Solve the equation $y'' + 3y' + 2y = \delta(t-1)$, $y(0)=0, y'(0)=0$.

26. Find the Laplace transform of $e^{-4t} \int_0^t t \sin 3t \, dt$.

27. Find the Laplace transform of $t^2 e^{-3t} \sin 2t$.

28. Find the inverse Laplace transform of $\frac{2(e^{-s} - e^{-3s})}{s^2 - 4}$.

29. Find the inverse of $\frac{5s+3}{(s-1)(s^2+2s+5)}$.

30. Find the Fourier series for $f(x)=|x|$ from $x=-\pi < x < \pi$ and deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}.$$

31. Using Fourier sine integral, Show that

$$\int_0^{\infty} \frac{1 - \cos \pi w}{w} \sin x w \, dw = \begin{cases} \frac{\pi}{2} & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$$

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15 marks**.

32. (a) State and prove convolution theorem of Laplace transforms

(b) Use convolution theorem to find the inverse Laplace transform of $\frac{1}{s^2(s-a)}$.

33. Find the Fourier series of the periodic function $f(x)$ of period 2, where $f(x)=\pi x$ in $0 \leq x \leq 2$ and deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

34. (a) Represent $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ as Fourier cosine integral

(b) Find the Fourier integral of $f(x) = \begin{cases} \pi - x & \text{if } 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$

35. (a) Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$. What is your inference?

(b) Find the Fourier transform of $f(x) = \begin{cases} |x| & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

(2 × 15 = 30 Marks)