

Reg. No. :

Name :

Fourth Semester B.Sc. Degree Examination, May 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 : MATHEMATICS IV – COMPLEX ANALYSIS, SPECIAL
FUNCTIONS AND PROBABILITY THEORY

(2019 Admission Regular)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They carry 1 mark each.

1. Define analytic function.
2. If $f(z)$ is analytic in a region R , then what we can say about the analyticity of $f^{iv}(z)$ in R .
3. Find $\int_C (z^2 + z + 1)dz$, where C is the circle $|z| = 5$.
4. Define residue.
5. Find $\Gamma(3)$.
6. Define Beta function.
7. Define sample space.

8. Define probability.
9. State Baye's formula.
10. Write down the probability density function of a Binomial distribution.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Distinguish between regular point and singular point of a function $f(z)$.
12. State a sufficient condition for analyticity of a function $f(z)$ in a region.
13. State Laurent's theorem.
14. Distinguish between pole and essential singular point.
15. Find the order of the pole at $z = 0$ of $\frac{\sin z}{z^5}$.
16. Find $\int_C \frac{z+1}{z-2} dz$, where C is the circle $|z| = 3$, oriented in the counter clockwise direction.
17. Find residue at the pole $z = 1$ of the function $f(z) = \frac{1}{(z-2)^2(z-1)}$.
18. Find residue at the pole $z = \infty$ of the function $f(z) = \frac{1}{z}$.
19. Find $\Gamma\left(\frac{2}{3}\right) \div \Gamma\left(\frac{8}{3}\right)$.
20. Find $\int_0^{\infty} x^2 e^{-x} dx$.

21. State recurrence relation for Gamma function.
22. Find the probability that a single card drawn from a shuffled deck of cards will be either a diamond or a king.
23. What is the probability of showing two heads and one tail, when three coins are tossed?
24. What is the probability of being able to form a two-digit number greater than 33 with the two numbers on the dice, when two dice are thrown?
25. Two students are working separately on the same problem. If the first student has probability $\frac{1}{2}$ of solving it and the second student has probability $\frac{3}{4}$ of solving it, what is the probability that at least one of them solves it?
26. If 1500 people each select a number at random between 1 and 500, what is the probability that 2 people elected the number 29?

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Evaluate $\int_C \frac{4z-3}{z(z-1)} dz$, where C is the circle $|z|=2$.
28. Find the residue of the function $f(z) = \frac{1}{(1+z^2)^2}$ at its singular points.
29. Expand $f(z) = \frac{1}{(z+2)(z+5)}$ as a Laurent series in the region $1 < |z| < 2$.
30. Prove that $\Gamma(p+1) = p\Gamma(p)$.

31. Evaluate $\int_0^{\infty} x^5 e^{-x^2} dx$.
32. Evaluate $\int_0^{\infty} \frac{x^3}{(1+x)^5} dx$.
33. Express the integral $\int_0^1 \frac{1}{\sqrt{1-x^3}} dx$ in beta function.
34. A club consists of 50 members. In how many ways can a president, vice-president, secretary, and treasurer be chosen? In how many ways can a committee of 4 members be chosen?
35. Find the probability of exactly 52 heads in 100 tosses of a coin using the binomial distribution and using the normal approximation.
36. Find the coefficient of x^8 in the binomial expansion of $(1+x)^{15}$.
37. Explain Joint Distributions.
38. Suppose that boxes of a certain kind of cereal have an average weight of 16 ounces and it is known that 70% of the boxes weigh within 1 ounce of the average. What is the probability that the box you buy weighs less than 14 ounces?

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each.

39. (a) State and prove Cauchy Integral formula
(b) Derive Cauchy Riemann equations.
40. Evaluate the Following integrals

(a) $\int_0^{2\pi} \frac{1}{5+3\cos\theta} d\theta$

(b) $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$

41. (a) Verify that the function $u(x,y) = x^3 - 3xy^2 - 5y$ is harmonic. Also find a harmonic conjugate of $u(x,y)$.

(b) Find the image of the given lines under the mapping $f(z) = z^2$

(i) $y = 2,$

(ii) $x = 0$

42. (a) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(b) Prove that $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$

43. (a) A preliminary test is customarily given to the students at the beginning of a certain course. The following data are accumulated after several years:

(i) 95% of the students pass the course, 5% fail.

(ii) 96% of the students who pass the course also passed the preliminary test.

(iii) 25% of the students who fail the course passed the preliminary test.

What is the probability that a student who has failed the preliminary test will pass the course?

(b) Find the number r such that the area under the normal distribution curve

$y = f(x)$ from $\mu = r$ to $\mu + r$ is equal to $\frac{1}{2}$.

44. (a) Derive Poisson probability density function.
- (b) There are 3 red and 5 black balls in one box and 6 red and 4 white balls in another. If you pick a box at random, and then pick a ball from it at random, what is the probability that it is
- (i) red
 - (ii) Black?
 - (iii) White?
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