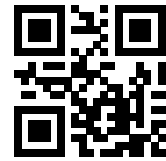


Reg No.:

Name :



U8352



# University of Kerala

First Semester Degree Examination, November 2024

Four Year Undergraduate Programme

Discipline Specific Course

## Mathematics

UK1DSCMAT102, Differentiation, Sequences and Series

Academic Level: 100-199

Time: 2 hours

Max. Marks: 56

Part A. 6 Marks. Time: 5 Minutes

Objective Type. 1 Mark Each. Answer all Questions

(Cognitive Level: Remember/Understand)

| Qn. No. | Question                                                                            | Cognitive Level | Course Outcome (CO) |
|---------|-------------------------------------------------------------------------------------|-----------------|---------------------|
| 1.      | State the product rule of differentiation.                                          | Remember        | CO2                 |
| 2.      | If $p < 1$ , $\sum_{k=1}^{\infty} \frac{1}{k^p}$ is convergent. True or false?      | Remember        | CO3                 |
| 3.      | Find $\frac{d}{dx}(x^\pi)$                                                          | Understand      | CO2                 |
| 4.      | Evaluate $\lim_{x \rightarrow 1} (x^7 - 2x^5 + 1)^{35}$ .                           | Understand      | CO1                 |
| 5.      | Write the limit comparison test                                                     | Remember        | CO3                 |
| 6.      | Is the series $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)$ convergent. Justify? | Remember        | CO3                 |

**Part B. 10 Marks.** Time:20 Minutes  
Two-Three sentences. 2 Marks Each. Answer all Questions  
(Cognitive Level: Remember/Understand/Apply)

| Qn. No. | Question                                                                                      | Cognitive Level | Course Outcome (CO) |
|---------|-----------------------------------------------------------------------------------------------|-----------------|---------------------|
| 7.      | State the Chain rule of differentiation                                                       | Remember        | CO2                 |
| 8.      | Write the ratio test                                                                          | Remember        | CO3                 |
| 9.      | Define convergence and divergence of a sequence.                                              | Remember        | CO3                 |
| 10.     | Find $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$ .                                        | Understand      | CO1                 |
| 11.     | Show that the sequence $\{\frac{10^n}{n!}\}_{n=1}^{\infty}$ is eventually strictly decreasing | Apply           | CO3                 |

**Part C. 16 Marks.** Time:35 Minutes  
Short-Answer. 4 Marks Each. Answer all Questions, choosing among options within each question.  
(Cognitive Level: Understand/Analyse/Apply)

| Qn. No. | Question                                                                                                                                                                                                                                                                                                                                                                                                                       | Cognitive Level | Course Outcome (CO) |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|---------------------|
| 12.     | A. Find the $n^{th}$ Taylor polynomial for $\frac{1}{x}$ about $x = 1$ and express it in sigma notation.<br>OR<br>B. Find the Maclaurin polynomials $p_0, p_1, p_2$ for $e^x$ .                                                                                                                                                                                                                                                | Understand      | CO1                 |
| 13.     | Let $y = x^2 + 1$<br><br>a) Find the average rate of change of y with respect to x over the interval $[3, 5]$ .<br><br>b) Find the instantaneous rate of change of y with respect to x when $x = -4$ .<br><br>OR<br>B. Find the equation of the tangent lines to the curve $y^2 - x + 1 = 0$ at the points $(2, -1)$ and $(2, 1)$ .                                                                                            | Understand      | CO1                 |
| 14.     | A. Let $f(x) = \begin{cases} \frac{1}{x+2}, & x < -2 \\ x^2 - 5, & -2 < x \leq 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$<br>Find (a) $\lim_{x \rightarrow -2} f(x)$ (b) $\lim_{x \rightarrow 0} f(x)$ (c) $\lim_{x \rightarrow 3} f(x)$ (d) $\lim_{x \rightarrow 1} f(x)$<br>OR<br>B. a) Find $\frac{d}{dx} [\sin(\sqrt{1 + \cos x})]$ .<br>b) Evaluate $\lim_{x \rightarrow 0+} \left( \frac{1}{x} - \frac{1}{\sin x} \right)$ . | Analyse         | CO2                 |
| 15.     | A.<br><br>1. Find the sum of the series $\sum_{k=1}^{\infty} \left( \frac{3}{4^k} - \frac{2}{5^{k-1}} \right)$<br><br>2. Find the limit of the sequence $\left\{ \frac{n}{e^n} \right\}_{n=1}^{\infty}$<br><br>OR<br>B. Determine whether the $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ converges.                                                                                                                   | Apply           | CO3                 |

**Part D. 24 Marks. Time:60 Minutes**

Long-Answer. 6 Marks Each. Answer all 4 Questions, choosing among options within each question.  
(Cognitive Level: Understand/Analyse/ Apply)

| Qn. No. | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Cognitive Level | Course Outcome (CO) |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|---------------------|
| 16.     | <p>A)</p> <ol style="list-style-type: none"> <li>Sketch the graph of the function <math>f(x) = 1 - 2^x</math> and find its domain and range.</li> <li>Find x such that <ol style="list-style-type: none"> <li><math>\log x = \sqrt{2}</math></li> <li><math>\ln(x+1) = 5</math></li> <li><math>5^x = 7</math></li> </ol> </li> </ol> <p><b>OR</b></p> <p>B)</p> <ol style="list-style-type: none"> <li>Find <math>\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}</math>.</li> <li>Evaluate <math>\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\sin x} \right)</math>.</li> </ol>    | Understand      | CO1                 |
| 17.     | <p>A) Solve <math>\frac{e^x - e^{-x}}{2} = 1</math> for x.</p> <p><b>OR</b></p> <p>B) Use Remainder Estimation Theorem to find an interval containing <math>x = 0</math> throughout which <math>f(x) = \cos x</math> can be approximated by <math>p(x) = 1 - \left(\frac{x^2}{2!}\right)</math> to three decimal place accuracy.</p>                                                                                                                                                                                                                                                             | Understand      | CO1                 |
| 18.     | <p>A)</p> <ol style="list-style-type: none"> <li>Find the Maclaurin polynomials <math>P_0, P_1, P_2</math> and <math>P_n</math> for <math>e^x</math>.</li> <li>Find the first three Taylor polynomials for <math>\ln x</math> about <math>x = 2</math>.</li> </ol> <p><b>OR</b></p> <p>B)</p> <ol style="list-style-type: none"> <li>State the Alternating series test.</li> <li>Using Alternating series test, show that the following series converge. a) <math>\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}</math> b) <math>\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}</math></li> </ol> | Analyse         | CO3                 |

| Qn.<br>No. | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Cognitive<br>Level | Course<br>Outcome<br>(CO) |
|------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|---------------------------|
| 19.        | <p>A)</p> <ol style="list-style-type: none"> <li>Find the area of triangle formed from the coordinate axes and the tangent line to the curve <math>y = 5x^{-1} - \frac{1}{5}x</math> at the point(5,0).</li> <li>Suppose that <math>s = f(t) = 1 + 5t - 2t^2</math> is the position function of a particle,where s is in meters and t is in seconds. Find the average velocities of the particle over the time intervals <math>[0, 2]</math> and <math>[2, 3]</math></li> </ol> <p><b>OR</b></p> <p>B) Using suitable test determine whether the following series converge or diverge.</p> <ol style="list-style-type: none"> <li><math>\sum_{k=3}^{\infty} \frac{(2k)!}{4^k}</math></li> <li><math>\sum_{k=2}^{\infty} \left( \frac{4k-5}{2k+1} \right)^k</math></li> <li><math>\sum_{k=1}^{\infty} \frac{1}{(\ln(k+1))^k}</math></li> </ol> | Apply              | CO2, CO3                  |