



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2018

First Degree Programme under CBCSS

PHYSICS

Elective Course

PY 1661.2 – Space Science

(2013 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions in a word or **one** or **two** sentences. **Each** question carries **one** mark.

1. What is Astronomy ?
2. What are the different types of galaxies based on their shape ?
3. What is dark matter ?
4. What is super cluster ?
5. What is the condition for supernova explosion ?
6. What are sun spots ?
7. Discuss the ionospheric electric fields.
8. Explain the structure of a Quasar.
9. What is Comet ?
10. What is relativistic cosmology ?

SECTION – B

Answer **any eight** questions **not** exceeding in a paragraph. **Each** question carries **two** marks.

11. Give a brief description of our galaxy-milky way.
12. Distinguish between a meteor and asteroid.
13. Write the sequence of stellar evolution.

E – 1644



14. Derive the expression for gravitational potential energy of a star.
15. What is solar wind ? What is its origin ?
16. What causes radiation belts around earth ? Mention their names.
17. Give short notes on neutron stars and super nova.
18. How a red giant become a white dwarf star ?
19. What causes sunspots ?
20. What do you mean by 'red shift' ? How these are useful in astrophysics ?
21. Give the composition of earth's atmosphere.
22. Define magneto tail and plasma sheet.

SECTION – C

Answer **any six** questions. **Each** question carries **four** marks.

23. Give the distribution of galaxies across the sky based on Hubble's law.
24. Give a brief description of solar activity.
25. Give the nomenclature and temperature profile of earth's atmosphere.
26. With neat diagram, explain the structure and composition of sun.
27. Discuss the solar wind interaction effects on earth's magnetic field.
28. Write a short note on Chapman-Ferraro closed magnetic sphere.
29. Give a brief note on expansion of universe.
30. What is meant by a) black holes b) white dwarfs.
31. What causes solar wind ? Give its important effects on earth's atmosphere.

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

32. Explain the types of galaxies based on different classifications.
33. Starting from the birth of a star, explain the classification of stars.
34. Give a detailed account of earth's magneto sphere and its effect on solar activity and earth's magnetic weather.
35. Describe the structure of Earth's atmosphere.



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Sixth Semester B.Sc. Degree Examination, April 2018

First Degree Programme Under CBCSS

CHEMISTRY

Core Course – X

CH 1641 : Organic Chemistry – II

(2013 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. Answer in **one** word to **maximum two** sentences.**Each** question carries **one** mark :

1. Draw the Zwitter ion form of glycine.
2. What is the product obtained when phenol is treated with Con.HNO_3 ?
3. What is the action of heat on anthranilic acid ?
4. Give any one method of preparation of coumarin.
5. What is the product of the reaction $2\text{CH}_3\text{CHO} \xrightarrow{\text{OH}^-}$?
6. What are anomers ?
7. What is the product obtained when glucose is acetylated ?
8. What is a prosthetic group ?
9. Give an example for a synthetic detergent.
10. What is special isoprene rule ?

(10×1=10 Marks)

SECTION – B

Short answer type. Answer **any 8** questions from the following.**Each** question carries **two** marks.

11. Phenol is a weaker acid than acetic acid. Explain.
12. Give any one method of preparations of catechol.



13. What happens when $\text{CH}_3\text{CH}=\text{CH}-\text{CHO}$ is reduced with NaBH_4 ?
14. Why is the α -hydrogen in aldehydes and ketones acidic.
15. How will you obtain cinnamic acid by knoevenagel reaction ?
16. Why are carboxylic acids much stronger than alcohols ?
17. What are epimers ? Give an example.
18. Draw the Haworth ring structure of α -D fructose.
19. What is denaturation of proteins ?
20. Write a note on stereochemistry of amino acids.
21. Write a note on chemistry of vision.
22. What is genetic code ?

(8×2=16 Marks)

SECTION - C

Short essay type. Answer **any 6** questions from the following.

Each question carries **four** marks.

23. What is Fries rearrangement. Explain the mechanism.
24. Explain the preparation and use of :
 - i) Picric acid
 - ii) Resorcinol
 - iii) Quinol.
25. What is witting reaction ? Explain the mechanism.
26. Explain with mechanism the oxidative cleavage of diols using lead tetra acetate and periodic acid reagent.
27. How is salicylic acid prepared ? How would you convert it into :
 - a) Phenol
 - b) Benzoic acid
 - c) Aspirin.



28. Explain the preparation, properties and structure of cellulose and starch.
29. Explain Sheehan method of peptide synthesis.
30. What is Wolff-Kishner reduction ? Give its mechanism.
31. Explain the classification of vitamins and represent the structure of vitamins A, B₁ and C. (6×4=24 Marks)

SECTION - D

Answer **any 2** questions. **Each** question carries **15** marks.

32. Explain the mechanism of following reactions :
 - i) Aldol condensation
 - ii) Perkin reaction
 - iii) Knoevenagel condensation
 - iv) Wolff-Kishner reduction.
33. Explain elaborately, the structure of glucose.
34. Explain elaborately, the structure elucidation and extraction of nicotine.
35. a) Explain any two, preparation and uses of :
 - i) Salicylic acid
 - ii) Tartaric acid
 - iii) Anthranilic acid
 - iv) Citric acid.

b) Explain :
 - i) Structure of RNA and DNA
 - ii) Replication of DNA.

(15×2=30 Marks)



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Sixth Semester B.Sc. Degree Examination, April 2018

First Degree Programme under CBCSS

Chemistry

Core Course – XII

CH 1643 – PHYSICAL CHEMISTRY – III

(2013 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. Answer in **one** word to maximum **two** sentences. **Each** question carries **one** mark.

1. For the reaction $A + B \rightarrow \text{Products}$, it is observed that on doubling the concentration of A only the rate of the reaction is doubled and also on doubling the concentration of B only the rate of the reaction is doubled. What is the rate equation ?
2. What is an acidic buffer ?
3. Explain the term half life of a reaction.
4. What is ionic product of water ?
5. What is efflorescence ?
6. Define the term components in phase equilibria.
7. Give an example for upper critical solution temperature.
8. What is Azeotropic mixture ?
9. The standard reduction potential of 3 elements are given below. Which of the following is the most suitable reducible agent ? i) – 3.04 V ii) – 1.90 V iii) 1.90 V.
10. What is Kohlrausch's law ? **(10×1=10 Marks)**



SECTION – B

Short answer type. Answer **any 8** questions from the following. **Each** question carries **two** marks.

11. Calculate the pH of :

a) 0.03 N H_2SO_4

b) 0.03 M H_2SO_4

12. Define consecutive and parallel reaction.

13. Explain solubility product with an example.

14. Derive the relation between K_p and K_c .

15. Explain the terms :

i) Triple point

ii) Eutectic point.

16. What is Phase rule ? Explain the terms.

17. Explain the principle of steam distillation.

18. Explain intermediate compound formation theory in catalysis.

19. What is Nernst equation ? Explain the terms.

20. What is Debye-Falkenhagen effect ?

21. What is Beer-Lambert law ?

22. Explain the terms :

i) Fluorescence

ii) Phosphorescence.

(8×2=16 Marks)

SECTION – C

Short essay type. Answer **any 6** questions from the following. **Each** question carries **four** marks.

23. The half life of first order decomposition of Nitramide is 2.1 hour at 150 °C.
 $\text{NH}_2\text{NO}_{2(aq)} \rightarrow 2\text{N}_2\text{O}_{(g)} + \text{H}_2\text{O}_{(l)}$ if 6.2 gm of NH_2NO_2 is allowed to decompose.
Find the time taken for nitramide to decompose 99 % ?



24. Derive the relation between K_h , K_w and K_a for the hydrolysis of salt of a weak acid and strong base.
25. Explain the phase diagram of water system.
26. State and explain Nernst Distribution law.
27. Write a note on anion reversible cells.
28. Explain the working of quinhydrone electrode.
29. Explain Michaelis-Menten law.
30. Explain the determination of transport number by Hittorf's method.
31. Define quantum yield. Explain the reason for very high and low quantum yields. (6×4=24 Marks)

SECTION – D

Answer **any 2** questions from the following. **Each** question carries **15** marks.

32. Explain different methods to determine the order of reaction.
 33. Derive Clausius – Clapeyron equation and explain its application.
 34. i) Explain the working of $H_2 - O_2$ fuel cell. What are its advantages ?
ii) Explain qualitatively $H_2 - Cl_2$ photochemical reaction.
 35. i) Explain potentiometric titration for acid base and redox reaction.
ii) A buffer solution contains 0.2 mole of NH_4OH and 0.5 mole of NH_4Cl per litre. Calculate the pH of the solution. The dissociation constant of NH_4OH is 1.8×10^{-5} . (2×15=30 Marks)
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Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2018
First Degree Programme under CBCSS and Career Related

CHEMISTRY
Elective Course
Common for CH 1661.4/IC 1661.4
Biochemistry
(2013 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Lipoproteins are classified based on their
2. Oxidation state of Iron in hemoglobin is
3. Largest blood cell is
4. Which ion is necessary for clotting of blood ?
5. Enzyme present in gastric juice is
6. Major component of urine is
7. Hemoglobin combines with CO_2 to form
8. BMR stands for
9. Mobile phase in paper chromatography is
10. Name of primary bile acid is (1×10=10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. What are the constituents of plasma ?
12. Explain GLC.
13. Comment on adult haemoglobin and fetal haemoglobin.

14. Differentiate between plasma and serum.

15. What is bile ?

16. Write note on enzyme mechanism.

17. What is Rf value ?

18. Write note on ultrafiltration.

19. Discuss the significance of BMR.

20. Write the renal threshold value of glucose.

21. How much calorie is obtained from 1g fat ?

22. What are the abnormal constituents of urine ?

(8×2=16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

23. Explain the interrelationship between Oxygen and Carbon dioxide transport.

24. How water level is maintained in the body ?

25. Explain the paper chromatography.

26. Explain the buffers in blood.

27. What is meant by co-operative effect ? Explain its significance.

28. Briefly describe ion exchange chromatography.

29. Write the composition and function of bile.

30. What is pulmonary respiration ?

31. Write short notes on TLC.

(6×4=24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

32. Explain the digestion and absorption of fat.

33. Write notes on plasma protein, albumin and globular proteins.

34. What are coagulation factors ? Explain the mechanism of coagulation.

35. Outline the digestive system in human and explain the function.

(15×2=30 Marks)



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Sixth Semester B.Sc. Degree Examination, April 2018

First Degree Programme under CBCSS

MATHEMATICS

Core Course – IX

MM 1641 : Real Analysis – II

(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of a function $f : [0, 1] \rightarrow \mathbb{R}$ that is discontinuous at every point of $[0, 1]$ but such that $|f|$ is continuous on $[0, 1]$.
2. Let $f : A \rightarrow \mathbb{R}$ for $A \subset \mathbb{R}$. State the sequential criterion for continuity of f at $a \in A$.
3. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and takes only rational values then f is constant. True or false ?
4. Evaluate $\lim_{x \rightarrow 0} \left[\frac{1 - \cos x}{x^2} \right]$.
5. State the Interior Extremum Theorem.
6. Let $f : (a, b) \rightarrow \mathbb{R}$ and $c \in (a, b)$. Explain what is meant by the statement, " f is differentiable at $x = c$ " ?
7. Suppose that f has an inverse and $f(2) = -4$, $f'(2) = 3/4$. If $g = 1/f^{-1}$, what is $g'(-4)$?
8. Find $F'(x)$ when F is defined on $[0, 1]$ by $F(x) = \int_0^{x^2} (1+t^3)^{-1} dt$.
9. If $f(x) = x^2$ for $x \in [0, 4]$, calculate the Riemann sum, where $P = \{0, 1, 2, 4\}$ and the tags are selected at the left end points of the sub-intervals.

10. Let $f \in \mathcal{R}[a, b]$ and define $F(x) = \int_a^x f$ for $x \in [a, b]$. Evaluate $S(x) = \int_x^{\sin x} f$ in terms of F .

SECTION – II

Answer **any 8** questions from among the questions **11 to 22**. These questions carry **2 marks each**.

11. Let $A, B \subseteq \mathbb{R}$ and let $f : A \rightarrow \mathbb{R}$ and $g : B \rightarrow \mathbb{R}$ be functions such that $f(A) \subseteq B$. If f is continuous at a point $c \in A$ and g is continuous at $b = f(c) \in B$, show that the composition $g \circ f : A \rightarrow \mathbb{R}$ is continuous at c .
12. Let I be an interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . If $a, b \in I$ and if $k \in \mathbb{R}$ satisfies $f(a) < k < f(b)$, show that there exists a point $c \in I$ between a and b such that $f(c) = k$.
13. Let I be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Show that the set $f(I)$ is a closed bounded interval.
14. Suppose that f is continuous on a closed interval $I = [a, b]$, that the derivative f' exists at every point of the open interval (a, b) , and that $f(a) = f(b) = 0$. Show that there exists at least one point c in (a, b) such that $f'(c) = 0$.
15. Let I be an interval and let $f : I \rightarrow \mathbb{R}$ be differentiable on I . Show that if f' is positive on I , then f is strictly increasing on I .
16. If f is differentiable on $I = [a, b]$ and if k is a number between $f'(a)$ and $f'(b)$, show that there is at least one point c in (a, b) such that $f'(c) = k$.
17. Let $n \in \mathbb{N}$ and let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^n$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$. For which values of n is f' continuous at 0 ? For which values of n is f' differentiable at 0 ?
18. Evaluate $\lim_{x \rightarrow \infty} (1 + 1/x)^x$.
19. Prove that $e^\pi > \pi^e$.
20. If $f \in \mathcal{R}[a, b]$ and $|f(x)| \leq M$ for all $x \in [a, b]$, show that $\left| \int_a^b f \right| \leq M(b-a)$.



21. If f is continuous on $[a, b]$, $a < b$, show that there exists $c \in [a, b]$ such that we have $\int_a^b f = f(c)(b - a)$.
22. Applying the fundamental theorem, show that there does not exist a continuously differentiable function f on $[0, 2]$ such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for $0 \leq x \leq 2$.

SECTION - III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Show by example that a function that is differentiable at every point of $\mathbb{R}$ need not have a continuous derivative.
24. State and prove the Caratheodory's theorem.
25. Let f be defined on $\mathbb{R}$ by
- $$f(x) = \begin{cases} x^2 + ax + b & \text{if } x \geq 0 \\ \sin x & \text{if } x < 0 \end{cases}$$
- Is it possible to find a, b such that f is differentiable on $\mathbb{R}$? If not, explain why; if yes, give the values of a, b .
26. Find the points of relative extrema of the function $f(x) = |x^2 - 1|$ for $-4 \leq x \leq 4$.
27. State and prove the Taylor's theorem.
28. Evaluate $\lim_{x \rightarrow 0} \frac{x^2 - \sin^2 x}{x^4}$.
29. If $f \in \mathcal{R}[a, b]$, show that f is bounded on $[a, b]$.
30. State and prove the general form of Integration by Parts for the Riemann integral.

31. Show that every continuous function is Riemann integrable.



SECTION – IV

Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each**.

32. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \rightarrow \mathbb{R}$ be strictly monotone and continuous on I . Show that the function g inverse to f is strictly monotone and continuous on $f(I)$.
33. State and prove the Newton's method to estimate a solution of an equation.
34. a) If $f : [a, b] \rightarrow \mathbb{R}$ is monotone on $[a, b]$, show that $f \in \mathcal{R}[a, b]$.
b) Let $K(x) = x^2 \cos(1/x^2)$ for $x \in (0, 1]$ and let $K(0) = 0$. Find $K'(x)$ and check whether $K' \in \mathcal{R}[0, 1]$.
35. a) Show that, in general, the indefinite integral need not be an antiderivative.
b) State and prove the fundamental theorem of calculus (second form).
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Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2018**First Degree Programme Under CBCSS****Mathematics****Core Course X****MM 1642 : LINEAR ALGEBRA****(2014 Admn. Onwards)**

Time : 3 Hours

Max. Marks : 80

SECTION – I**All the first 10 questions are compulsory. Each question carries 1 mark.**1. For which choice of x will the following pairs be linearly dependent ?

a) $\begin{pmatrix} x \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

b) $\begin{pmatrix} x \\ x^2 \end{pmatrix}, \begin{pmatrix} 1 \\ x \end{pmatrix}$

2. Show that the parametric equation $X = V + t(U - V)$ represents the line through U and V if U and V are any two vectors which are not equal.3. Find the angle between the vectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.4. Find the distance from the $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ to the line $y = 2x$.5. Let S be the transformation which assigns to each vector X , the reflection of X in the line along the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. If $X = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$, find $S(X)$.6. Let A be the transformation with matrix $\begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$. Find the image of the line along the vector $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ under A .



7. Define eigenvalue and eigenvector of a linear transformation.
8. Define Isometry transformation.
9. Classify the quadric surface $4x^2 - 9y^2 + z^2 + 36 = 0$.
10. Find a non-zero vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ in $\mathbb{R}^3$ which is orthogonal to each of the vectors $\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Find the characteristic polynomial of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix}$.
12. Show that the transformation with matrix $\begin{pmatrix} 2 & 0 & 0 \\ 4 & -1 & 0 \\ 2 & 3 & -1 \end{pmatrix}$ is invertible.
13. Let P denote the projection on the line along U . Find $m(P)$.
14. Let T be a length preserving transformation and the matrix of T is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.
Show that T is a rotation when the determinant is 1 and a reflection when the determinant is -1 .
15. Prove that $R_{\frac{\pi}{2}}$ has no eigenvalue.
16. In terms of coordinates of $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, calculate the images of $R_{\frac{\pi}{2}}^1$ and $R_{\frac{\pi}{2}}^3$.
17. If $m(T) = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix}$ and $m(S) = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 3 & 1 \end{pmatrix}$, then show that $m(ST) \neq m(TS)$.

18. Define Shear and Permutation matrices.
19. Let S and T be linear transformation of R^3 such that $ST = I$. Prove that $TS = I$.
20. Find all eigenvalues of the matrix $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$.
21. Find an equation in new coordinates of the form $\lambda_1 u^2 + \lambda_2 v^2 + \lambda_3 w^2 = 1$ for the quadric surface $-4x^2 + 2y^2 + 3yz + 2z^2 = 1$.
22. If T is an isometry of R^3 , then prove that T has 1 or -1 as an eigenvalue.

SECTION – III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. Let A be a linear transformation and let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be its matrix. If $|ad - bc| > 0$, then show that A preserves the orientation.
24. If $\vec{A} \times \vec{B} = 0$ then prove that $\vec{A}$ and $\vec{B}$ are linearly dependent.
25. Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be an arbitrary matrix. Prove that there exists elementary matrices e_1, e_2, e_3, e_4 and diagonal matrix $\begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$ such that $e_1 e_2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} e_3 e_4 = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$.
26. Find the inverse of $R_{\frac{3\pi}{2}}$.
27. Find all solution of the non-homogeneous system

$$\begin{aligned} x + 2y - 4z &= -4 \\ 2x + 5y - 9z &= -10 \\ 3x - 2y + 3z &= 11. \end{aligned}$$
28. Prove that an isometry of R^3 preserves the dot product.
29. Let m be a symmetric 3×3 matrix and let M be the corresponding linear transformation. If X_1 and X_2 are the eigenvectors corresponding to the distinct eigenvalues t_1 and t_2 then show that $X_1 \cdot X_2 = 0$.



30. Find a non-zero vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ in $\mathbb{R}^3$, which is orthogonal to each of the vectors $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

31. Classify the quadric surface $x^2 + 3y^2 + z^2 + 2xy + 2xz + 2yz - 2x + 4y + 2z + 12 = 0$.

SECTION - IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. Let $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$. Find all eigenvalues and corresponding eigenvectors. Also find the matrices P and D such that $P^{-1}AP = D$ is diagonal.

33. State and prove Spectral theorem in $\mathbb{R}^3$.

34. a) Let T be a linear transformation of $\mathbb{R}^3$ which satisfies the condition : $T(X) = 0$ implies that $X = 0$. Prove that T has an inverse.

- b) Solve the system

$$x_1 - 2x_2 + 3x_3 = 0$$

$$x_1 + x_2 + x_4 = 0$$

$$4x_3 - x_4 = 0$$

- c) Show that $(e_{12}^s)^{-1} = e_{12}^{-s}$.

35. a) Let T be an orientation preserving linear transformation of $\mathbb{R}^3$. If Π is any parallelepiped then prove that $\text{volume}(T(\Pi)) = |\det(m(T))|(\text{volume}(\Pi))$.
- b) Given three points in the plane (x_1, y_1) , (x_2, y_2) , (x_3, y_3) which are not collinear, show that there exists a circle which passes through the three points.



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2018

First Degree Programme under CBCSS

MATHEMATICS

Core

MM 1643 : Complex Analysis – II

(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first 10 questions are compulsory. Each carries 1 mark.

1. Define power series.
2. Find the circle of convergence of the power series $\sum_{n=0}^{\infty} z^n$.
3. Find the set of all singularities of $f(z) = |z|^2$.
4. Define essential singularity of a complex function.
5. Find the poles of $\frac{z + 2z^2}{\sin z}$.
6. Find the residue of $f(z) = \frac{z - \sin z}{z}$ at $z = 0$.
7. Find the order of zero of $f(z) = (ze^z - ze^z \cos z) - (z - z \cos z)$ at $z = 0$.
8. Find the Cauchy principal value of $\int_{-\infty}^{\infty} x dx$.
9. If $f(z)$ is complex function having a pole of order 5 at $z = 0$. Then what type of singularity have the function $z^5 f(z)$?
10. What is the principal part of the Laurent's series expansion of $\frac{e^z}{z}$ at $z = 0$.

P.T.O.



SECTION - B

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. If f is analytic in $D = D(r)$ and if the closed rectangle R and the point a are both contained in D and C represents the boundary of R , then prove that
$$\int_C \frac{f(z) - f(a)}{z - a} dz = 0.$$
12. State Cauchy's integral formula.
13. Evaluate $\int_C \frac{z^2 + 1}{(z^2 - 1)^2} dz$, where C is the circle $|z - 1| = 1$ taken in counter clockwise direction.
14. Evaluate $\int_C z^2 \sin\left(\frac{1}{z}\right) dz$, where C is the circle $|z| = 1$ oriented in counter clockwise direction.
15. Find the residues of $\frac{z + 1}{z^2(z - 2)}$ at its poles.
16. If f is a complex function which is analytic in and on positively oriented circle C_R centered at z_0 with radius R and if M_R denotes the maximum of $|f(z)|$ on C_R then show that $|f^n(z_0)| \leq \frac{n! M_R}{R^n}$.
17. Find $\int_C \frac{1}{z^2 + 9} dz$, where C is the circle $|z - i| = 3$ oriented positively.
18. Find power series representation of $\sinh z$ in powers of z .
19. Find the residue of $f(z) = \tanh z$ at $z = \frac{\pi}{2}$.
20. If p and q are two complex functions, which are analytic at a point z_0 so that $p(z_0) \neq 0$ and q has a zero of order m at z_0 . Show that $\frac{p(z)}{q(z)}$ has a pole of order m at z_0 .
21. Evaluate $\int_0^\infty \frac{dx}{x^2 + 1}$.
22. Show that $\sum_{n=0}^\infty \frac{1}{n^2} = \frac{\pi}{6}$.



SECTION - C

Answer **any 6** questions from among the questions **23 to 31**. These questions carry **4 marks each** :

23. Let C be a simple closed contour, described in the positive sense in the z -plane and if $g(z) = \int_C \frac{s^3 + 2s}{(s-z)^3} ds$, then find the value of $g(z_0)$ for z_0 interior to C and exterior to C .

24. Evaluate $\int_C \frac{dz}{(z^2+1)(z^2-4)}$, where C is the circle $|z| = \frac{3}{2}$ oriented in counter clockwise direction.

25. State and prove Cauchy's residue theorem.

26. Evaluate $\int_C \frac{e^z}{(z-1)^2(z^2+9)} dz$, where C is the circle $|z-3i| = 1$ oriented in counter clockwise direction.

27. Evaluate the improper integral $\int_0^\infty \frac{x^2 dx}{(x^2+1)(x^2+4)}$.

28. Evaluate $\int_{-\infty}^\infty \frac{\cos 3x dx}{(x^2+1)^2}$.

29. Find the residue of $f(z) = \frac{1}{z(z-2)^4}$ at its poles and hence find $\int_C f(z) dz$ where C is the positively oriented circle $|z-2| = 1$.

30. Show that $\sum_{n=1}^\infty \frac{1}{n^2+1} = \frac{-1}{2} + \frac{\pi}{2} \left(\frac{e^{2\pi}+1}{e^{2\pi}-1} \right)$.

31. Show that $\sum_{n=0}^n \binom{3n}{n} \frac{1}{8^n} = \frac{5+3\sqrt{5}}{5}$.



SECTION - D

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Evaluate $\int_C \frac{dz}{4z^2 + 1}$ where C is the circle $|z| = 1$ oriented in counter

clockwise direction.

b) Evaluate $\int_C \frac{2z^2 - z - 2}{(z - 2)^4} dz$ where C is the circle $|z| = 3$ oriented in counter clockwise direction.

c) Show that $\int_C \frac{e^{az}}{z^{n+1}} dz = \frac{2a^n \pi i}{n!}$ where C is the circle $|z| = 1$ oriented in counter clockwise direction.

33. a) Prove that an isolated singular point z_0 of a function f is a pole of order m if and only if $f(z)$ can be written in the form $f(z) = \frac{\phi(z)}{(z - z_0)^m}$ where $\phi(z)$ is analytic and nonzero at z_0 .

b) Show that $\int_C e^{\left(z + \frac{1}{z}\right)} dz = 2\pi i \left(\sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \right)$ where C is the circle $|z| = 1$ oriented in counter clockwise direction.

34. a) Evaluate the integral $\int_0^{\infty} \frac{x^2}{x^6 + 1} dx$.

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta}$ where $-1 < a < 1$.

35. a) Prove that if f is complex function having only finite number of poles $\{z_k\}$, then $\sum_{n=1}^{\infty} f(n) = - \sum_k \text{Res}(f(z) \pi \cot \pi z, z_k)$.

b) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2 + a^2)(x^2 + b^2)}$ where $a > b > 0$.