

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. Diagonalize the symmetric matrix $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$.

33. (a) Find for what values of a and b , the equations

$$x + 2y + 3z = 6$$

$$3x + 3y + 5z = 9$$

$$2x + 5y + az = b$$

have

(i) no solution

(ii) a unique solution

(iii) more than one solution?

(b) Find the value of k for which the equations

$$2x + 3y + 4z = 0$$

$$x + 2y - 5z = 0$$

$$3x + 5y - kz = 0$$

have a non-trivial solution.

34. Verify Greens theorem for $\int_C (xy + y^2) dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

35. (a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$

(b) Solve $\frac{dy}{dx} + \frac{x}{1-x^2} y = x\sqrt{y}$.

(2 × 15 = 30 Marks)

(Pages : 4)

S – 3421

Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 MATHEMATICS III — LINEAR ALGEBRA, SPECIAL FUNCTIONS AND CALCULUS

(2021 Admission onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

(All the first **ten** questions are compulsory. They carry **1** mark each).

1. Write the differential equation corresponding to $y = a \cos nx$.

2. State Cayley - Hamilton theorem.

3. Find the rank of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

4. Which one of the following matrices is in the reduced echelon form?

$$\begin{bmatrix} 1 & 2 & 3 & 7 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 5 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Verify that $y=e^{-2x}$ is a solution of $y''+y'-2y=0$.
6. Find the particular integral of $y''+9y=e^{2x}$.
7. Define an inverse square field.
8. Determine whether the force field $F=4y\mathbf{i}+4x\mathbf{j}$ is a conservative or not.
9. If σ any closed surface enclosing a volume V and $r=xi+yj+zk$, prove that $\iiint_{\sigma} r \cdot n dS = 3V$.
10. Find $\beta(1,1)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Show that a square matrix A can be expressed as a sum of two matrices of which one is symmetric and the other is Skew symmetric.
12. Find A and B if $A+B=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $A-B=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.
13. If C is the straight line path from $(0,0,0)$ to $(1,1,1)$, then evaluate $\int_C dx + 2dy + 3dz$.
14. Show that $x=a\cos nt$ is a solution of the differential equation $\frac{d^2x}{dt^2} + n^2x=0$.
15. State Greens theorem including all hypotheses.
16. Solve $\frac{dy}{dx} + y \tan x = \cos x$.
17. Solve $(y''+y'+1)^2=0$.
18. Find the work done in moving a particle in the force field $F=3x^2\mathbf{i}+(2xz-y)\mathbf{j}-z\mathbf{k}$ from $t=0$ to $t=1$ along the curve $x=2t^2$, $y=t$, $z=4t^3$.
19. If $A=\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, find A^2 .

20. Find the sum and product of eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
21. If $A=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, find its characteristic equation of A .
22. Find the outward flux of the vector field $F(x,y,z)=2xi+3yj+z^2k$ across the unit cube $x=0, y=0, z=0, x=1, y=1, z=1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

23. Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 4 & -5 \\ 1 & -2 \end{bmatrix}$.
24. Using Cayley Hamilton theorem evaluate A^{-1} given $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.
25. Find the orthogonal trajectories of the family of curves $x^2-y^2=c^2$.
26. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.
27. Solve $(x+y)\frac{dy}{dx} = y-x$.
28. Find the number of solutions of the following system of equations $2x+6y+11=0$
 $4x+14y-z-7=0$
 $6y-3z+2=0$
29. Evaluate by stokes theorem $\oint_C (e^x dx + 2ydy - dz)$, where C is the curve $x^2+y^2=4, z=2$.
30. Using Gauss's divergence theorem evaluate $\iiint_S F \cdot n ds$ for $F=x^2\mathbf{i}+y^2\mathbf{j}+z^2\mathbf{k}$ taken over the region V of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.
31. Show that $\beta(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

(6 × 4 = 24 Marks)