



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2019

First Degree Programme under CBCSS

MATHEMATICS

Core Course – XII

MM 1644 : Abstract Algebra – II

(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are **compulsory**. Each carries 1 mark.

1. Find $\phi(25)$ for the homomorphism $\phi : \mathbb{Z} \rightarrow \mathbb{Z}_7$ such that $\phi(1) = 4$.
2. How many homomorphisms are there of $\mathbb{Z}$ into $\mathbb{Z}$?
3. Find the order of the factor group $(\mathbb{Z}_4 \times \mathbb{Z}_2) / \langle (2, 1) \rangle$.
4. The trivial subgroup $N = \{0\}$ of $\mathbb{Z}$ is a normal subgroup. Compute $\mathbb{Z}/\{0\}$.
5. The image of a group of 6 elements under a homomorphism may have 12 elements. True or False.
6. Compute the product $(-3, 5)(2, -4)$ in the ring $\mathbb{Z}_4 \times \mathbb{Z}_{11}$.
7. Find all units in the ring $\mathbb{Z} \times \mathbb{Z}$.
8. Find the characteristic of the ring $\mathbb{Z}_3 \times \mathbb{Z}_3$.
9. Using Fermat's theorem, find the remainder of 3^{47} when it is divided by 23.
10. A ring homomorphism $\phi : R \rightarrow R'$ carries ideals of R into ideals of R' . True or False.

P.T.O.



SECTION - II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Show that a group homomorphism $\phi: G \rightarrow G'$ is a one-to-one map if and only if $\text{Ker}(\phi) = \{e\}$.
12. Let H be a normal subgroup of G . Then show that $\gamma: G \rightarrow G/H$ given by $\gamma(x) = xH$ is a homomorphism with kernel H .
13. Does there exist a nontrivial homomorphism $\phi: \mathbb{Z}_3 \rightarrow \mathbb{Z}$? If yes, give an example. If not, explain why that is so.
14. Show that any group homomorphism $\phi: G \rightarrow G'$ where $|G|$ is a prime must either be the trivial homomorphism or a one-to-one map.
15. Show that a factor group of a cyclic group is cyclic.
16. Let $(R, +)$ be an abelian group. Show that $(R, +, \cdot)$ is a ring if we define $ab = 0$ for all $a, b \in R$.
17. Are the fields $\mathbb{R}$ and $\mathbb{C}$ isomorphic? Justify your answer.
18. In the ring $\mathbb{Z}_n$, show that the divisors of 0 are precisely those nonzero elements that are not relatively prime to n .
19. Show that 1 and $p - 1$ are the only elements of the field $\mathbb{Z}_p$ that are their own multiplicative inverse.
20. Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$ and having derivatives of all orders. Differentiation gives a map $\delta: F \rightarrow F$ where $\delta(f(x)) = f'(x)$. Is δ a homomorphism? Why?
21. Show that each homomorphism from a field to a ring is either one to one or maps everything onto 0.
22. Show that if R is a ring with unity and N is an ideal of R such that $N \neq R$, then R/N is a ring with unity.



SECTION - III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. Let $\phi: G \rightarrow G'$ be a group homomorphism. Show that if $|G|$ is finite, then $|\phi[G]|$ is finite and is a divisor of $|G|$.
24. Show that if a finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G .
25. Show that an intersection of normal subgroups of a group G is again a normal subgroup of G .
26. Show that if U is the collection of all units in a ring $(R, +, \cdot)$ with unity, then $(U, \cdot)$ is a group.
27. Show that every finite integral domain is a field.
28. Find all positive integers n such that $\mathbb{Z}_n$ contains a subring isomorphic to $\mathbb{Z}_2$.
29. Find all solutions of the congruence $155x \equiv 75 \pmod{65}$.
30. Let R be a commutative ring with unity of prime characteristic p . Show that the map $\phi_p: R \rightarrow R$ given by $\phi_p(a) = a^p$ is a homomorphism.
31. A ring R is a Boolean ring if $a^2 = a$ for all $a \in R$. Show that every Boolean ring is commutative.

SECTION - IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) Prove or disprove : If d divides the order of G , then there must exist a subgroup H of G having order d . 10
- b) Let ϕ be a homomorphism of a group G into a group G' . If K' is a subgroup of G' , then show that $\phi^{-1}[K']$ is a subgroup of G . 5



33. a) Let $\phi: G \rightarrow G'$ be a homomorphism with kernel H and let $a \in G$. Prove the set $\{x \in G \mid \phi(x) = \phi(a)\} = Ha$. 5
- b) Let H be a normal subgroup of G . Show that the cosets of H form a group G/H under the binary operation $(aH)(bH) = (ab)H$. 5
- c) Show that if H and N are subgroups of a group G , and N is normal in G , then $H \cap N$ is normal in H . Show by an example that $H \cap N$ need not be normal in G . 5
34. a) An element a of a ring R is idempotent if $a^2 = a$. Find all idempotents in the ring $\mathbb{Z}_6 \times \mathbb{Z}_{12}$. 5
- b) Show that the unity element in a subfield of a field must be the unity of the whole field. 5
- c) Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$. 5
35. a) Show that a division ring contains exactly two idempotent elements. 5
- b) Show that the characteristic of a subdomain of an integral domain D is equal to the characteristic of D . 5
- c) Show that $2^{11 \cdot 2^{13}} - 1$ is not divisible by 11. 5

St. Xavier's College



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MATHEMATICS

Core Course

MM 1642 : Linear Algebra

(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. Each carries **1** mark.1. Find a vector perpendicular to $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.2. Find the projection of $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ on line along $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.3. A is the transformation with matrix $\begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$. Find the image of the line along $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ under A.4. Calculate $R_\pi(X)$, if $X = \begin{pmatrix} x \\ y \end{pmatrix}$.5. Let P be the projection to the x_1x_2 plane and Q be the projection to the x_2x_3 plane. Find PQ.6. Find the distance from the point $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ to the plane $x_1 + x_2 + x_3 = 0$.

7. Define a linear transformation in 3-space.

8. Define cross product of two vectors in 3-space.

9. True or False : If T is the rotation about an axis then $\det(T) = 1$.10. Define projection to the hyperplane perpendicular to $U \neq 0$. (10×1=10 Marks)

P.T.O.



SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. If B is the reflection in the x – axis and A is the reflection in the y – axis, show that $AB = R_\pi$.

12. Let J_k be the transformation with matrix $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$. Show that $J_k J_m = J_{k+m}$.

13. If A and B are two transformations in 2-space then $\det(BA) = \det(B) \det(A)$.

14. Let C be the linear transformation in 2-space such that C has eigen values 0 and 1. Show that $C^2 = C$.

15. Show that in 3-space $A \times B = -B \times A$.

16. Show that square of symmetric matrix is symmetric.

17. Find a real number t such that $\begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}$ orthogonal to $\begin{bmatrix} t \\ -t \\ 1-t \\ 2t-1 \end{bmatrix}$.

18. Let m be an orthogonal matrix. Show that m^* is also orthogonal.

19. If T is an isometry of $\mathbb{R}^3$, then which are the eigen values of T ?

20. Let T be the linear transformation with matrix $m(T) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Show that $T^2 = 0$ and find all eigen values of T.

21. Show that if a scalar multiple of one row of a matrix is added to another row, the determinant is unchanged.

22. Show that $(e_{32}^s)^{-1} = e_{32}^{-s}$.

(8×2=16 Marks)



SECTION – III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. Find $S(X)$ if S is the perpendicular which assigns to each vector X the reflection of X in the line along the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

24. Prove that "Let A be a linear transformation and let L be a straight line. Then the image of L under A is either a straight line or a single point".

25. Prove that "Let A be a linear transformation with matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} > 0$, then A preserves orientation. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} < 0$ then A does not preserve orientation".

26. Find the n^{th} power of the matrix $\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$ for $n = 1, 2, 3, \dots$

27. Let $R_{\frac{\pi}{2}}$ be rotation by $\frac{\pi}{2}$. Find $\exp(R_{\frac{\pi}{2}})$.

28. Prove that "If A, B, C are three linearly independent vectors in $\mathbb{R}^3$, then every vector Y in $\mathbb{R}^3$ can be uniquely expressed as a linear combination of A, B and C ".

29. Solve the system of linear equations.

$$x + y + 2z = 3$$

$$x - y - 2z = -1$$

$$2x + 3y - z = 2$$

30. Classify the quadratic surface $\sum: x^2 + 2xy - 2xz = 1$.

31. Show that the vectors $\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix}, \begin{bmatrix} -y \\ x \\ -v \\ u \end{bmatrix}, \begin{bmatrix} u \\ -v \\ -x \\ y \end{bmatrix}, \begin{bmatrix} v \\ u \\ -y \\ -x \end{bmatrix}$ are orthogonal and all have the same length.

(6×4=24 Marks)