



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2019

First Degree Programme under CBCSS

MATHEMATICS

Core Course

MM 1641 : Real Analysis – II

(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are **compulsory**. Each carries 1 mark.

1. The function $g: \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = 2x$ for x rational, and $g(x) = x + 3$ for x irrational is continuous at $x =$ _____
2. Give an example for a function on $[0, 1]$ that is discontinuous at every point of $[0, 1]$ but $|f|$ is continuous on $[0, 1]$.
3. Find the points at which the function $f(x) = |x| + |x + 1|$ is not differentiable.
4. Using L'Hospital's Rule, find $\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$.
5. Define a convex function on an interval $I \subseteq \mathbb{R}$.
6. Let $g(x) = |x^3|, x \in \mathbb{R}$. Find $g'(x)$ for $x \neq 0$.
7. The norm of the partition $P = (0, 1.5, 2, 3.4, 4)$ is _____
8. If $F(x) = \frac{1}{2}x^2$ for all $x \in [a, b]$, is the antiderivative of f on $[a, b]$, evaluate $\int_a^b f$.
9. Define a step function.
10. If $J = [c, d]$ is a subinterval of $[a, b]$ and $\phi_J(x) = 1$ for $x \in J$ and $\phi_J(x) = 0$, elsewhere in $[a, b]$, then evaluate $\int_a^b \phi_J$.



SECTION - II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Show that the Dirichlet's function defined on $\mathbb{R}$ by, $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$ is not continuous at any point on $\mathbb{R}$.
12. If $I = [a, b]$ is a closed and bounded interval and $f : I \rightarrow \mathbb{R}$ is continuous on I , prove that f is bounded on I .
13. Show that the function $f(x) = \sin x$ is continuous on $\mathbb{R}$.
14. If $m \in \mathbb{Z}$, $n \in \mathbb{N}$ and $x > 0$, prove that $x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}}$.
15. If f and g are differentiable at c , show that $f g$ is differentiable at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.
16. Find $f'(0)$, if $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$.
17. If $f : I \rightarrow \mathbb{R}$ has a derivative at $c \in I$, show that f is continuous at c .
18. Explain briefly the tagged partition of a closed interval $[a, b]$.
19. If $f, g \in R[a, b]$ show that $f + g \in R[a, b]$.
20. State and prove boundedness theorem for Riemann integral.
21. Show that the set Q_1 of rational numbers in $[0, 1]$ is a null set.
22. If c is an interior point of an interval I at which $f : I \rightarrow \mathbb{R}$ has an extremum and if $f'(c)$ exists, then prove that $f'(c) = 0$.

SECTION - III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. State and prove Maximum Minimum Theorem.
24. State and prove Caratheodry Theorem.



25. If $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 1, & \text{if } 1 \leq x \leq 2 \end{cases}$. Show that $f \in R[0, 2]$ and evaluate the integral.
26. State and prove squeeze theorem.
27. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \rightarrow \mathbb{R}$ be monotone on I . Then prove that the set of points $D \subseteq I$ at which f is discontinuous is a countable set.
28. State and prove Rolle's Theorem.
29. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = x + 2x^2 \sin\left(\frac{1}{x}\right)$, for $x \neq 0$ and $g(0) = 0$.
• Show that g is not monotonic in any neighborhood of 0.
30. Prove that $1 - \frac{1}{2}x^2 \leq \cos x$ for all $x \in \mathbb{R}$.
31. If F, G are differentiable on $[a, b]$, and $f = F', g = G'$ belongs to $R[a, b]$, then prove that $\int_a^b fG = FG \Big|_a^b - \int_a^b Fg$.

SECTION - IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) State and prove location of roots theorem.
b) State and prove continuous inverse theorem.
33. a) State and prove Cauchy criterion for Riemann integrability.
b) If $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then prove that $f \in R[a, b]$.
34. a) State and prove additivity theorem.
b) If f and g are Riemann Integrable prove that fg is Riemann Integrable.
35. a) Let I be an open interval and let $f : I \rightarrow \mathbb{R}$ have a second derivative on I .
Then prove that f is a convex function if and only if $f''(x) \geq 0$ for all $x \in I$.
b) Use Newton's Method to find an approximate value of $\sqrt{2}$.
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