

Reg. No. :

Name :

Fourth Semester B.A. Degree Examination, May 2021

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Economics

MM 1431.5 – MATHEMATICS FOR ECONOMICS – IV

(2019 Admission Regular)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer all questions. Each question carries 1 mark.

1. Define a differential equation.
2. What do you mean by the degree of a differential equation?
3. Show that $y = ae^{nx}$ is an integral of $\frac{dy}{dx} = n y x^{n-1}$.
4. Is $\frac{dy}{dx} = \frac{x - 2y}{2x - y}$ a homogeneous differential equation.
5. What is the integrating factor for a linear differential equation, $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone?

6. Give the general form of a linear differential equation of order two with constant coefficients.
7. Write the associated equation of $3 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 2y = ae^x$.
8. What do you mean by complementary function of a linear differential equation?
9. If $y = a^x$ satisfies the equation $\frac{dy}{dx} = y$. Find a .
10. If marginal cost is $\frac{dC}{dx} = 2x + .5$, find the total cost $C(x)$.

(10 × 1 = 10 Marks)

PART – B

Answer any eight questions. Each question carries 2 marks.

11. Give an example of a second order and first degree differential equation.
12. Find the order and degree of $y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.
13. Find the differential equation for $y = cx + c - c^3$.
14. Solve $(1 + x) \frac{dy}{dx} = 2y$.
15. Show that the equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is variable separable.

16. Solve $\frac{dy}{dx} = e^{x+y}$.
17. If $\frac{dy}{dx} = \frac{x+y+3}{2x-2y+5}$ and $z = x - y$, evaluate $\frac{dz}{dx}$.
18. What is the integrating factor for $\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$?
19. Show that $y = \sin ax$ satisfies $\frac{d^2y}{dx^2} + a^2y = 0$.
20. Write the working formula for complementary function if the auxiliary equation of a second order linear equation has two distinct real roots.
21. Solve $(D^2 + 5D + 4)y = 0$.
22. How can we find the particular integral for the linear differential equation $f(D)y = X$, when $X = \cos ax$?
23. Find the particular integral for $(D^2 + D + 5)y = e^{-2x}$.
24. Give the definition of Euler equations.
25. Show that if y changes with respect to x according to the rule $\frac{dy}{dx} = -\frac{my}{x}$, then x and y are related by $y = \frac{a}{x^m}$.

26. Let Π is the total cost of an output x , $\frac{d\Pi}{dx}$, is the marginal cost and $\frac{\Pi}{x}$ is the average cost. Show that if marginal cost equals average cost, then Π is a constant multiple of x .

(8 × 2 = 16 Marks)

PART – C

Answer any six questions. Each question carries 4 marks.

27. Find the differential equation for $x^2 + y^2 + 2ax = 0$.
28. Find the differential equation of a family of lines touching the circle $x^2 + y^2 = a^2$, given by $y = mx + a\sqrt{1+m^2}$.
29. Solve $3(x-1)dy = 5(y+6)dx$.
30. Change the homogeneous equation $(x+y)dx + (x-y)dy$ into variable separable form.
31. Find the integrating factor of $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$.
32. Solve $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$.
33. Solve $D^4 - 81 = 0$.
34. Solve $(D^2 + 6D + 4)y = 0$.

35. Find the General solution of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x$.
36. Find the Complementary Function for $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$.
37. If $R = \frac{x-2}{y-3}$ is the marginal cost of Y for X, then find the total cost function.
38. Explain about the solution of Domar's Capital Expansion Model, through differential equations.

(6 × 4 = 24 Marks)

PART - D

Answer any two questions. Each question carries 15 marks.

39. (a) Find the differential equation having $y = ax^2 + bx$ as its general solution.
- (b) Find the differential equation having $y = Ae^{5x} + Be^{3x}$ as its general solution.
40. Define variable separable equations and solve
- (a) $x(1 + y^2) dx + y(1 + x^2) dy = 0$
- (b) $ye^{2x} dx - (1 + e^{2x}) dy = 0$.
41. (a) Define Homogeneous differential equations with an example.
- (b) Solve $(x^2 + y^2) dx = 2xy dy$.

42. Find the solution of $\frac{d^2y}{dx^2} + 4y = 8 \cos 2x$, given that $y = 0$ and $\frac{dy}{dx} = 0$, where $x = 0$.

43. Solve $(x^4 D^3 + 2x^3 D^2 - x^2 + x) y = 1$.

44. Define Marginal rate of substitution of Y for X. If $R = -\frac{3x + 5y}{4x + 6y}$ is the marginal rate of substitution of Y for X, then find its utility function, by solving the above differential equation.

(2 × 15 = 30 Marks)
