



Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, July 2017
First Degree Programme Under CBCSS
COMPLEMENTARY COURSE FOR MATHEMATICS
ST 1231.1 : Random Variables and Analysis of Bivariate Data
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions, **each** carries **one** mark :

1. Comment on the statement : If the sample space countable all the random variables defined on it is discrete.
2. When do you say that two random variables are independent ?
3. Let X be a random variable taking values 0, 1, 2 and 3 with probabilities $1/8$, $3/8$, $3/8$ and $1/8$; find the distribution function of X .
4. Find c if $f(x) = cx^3$, $0 < x < 1$ is a probability density function.
5. Define expectation of discrete and continuous random variable.
6. Define probability generating function of a random variable. Will it always exists ?
7. Justify the name moment generating function.
8. Why there are two regression lines ?
9. Explain need of rank correlation.
10. Show that independent random variables are uncorrelated. (10×1=10 Marks)



SECTION - B

Answer **any 8** questions, **each** carrying **2** marks :

11. Show that a distribution function is nondecreasing.

12. Examine if $F(x) = 0, x < 4$
 $= 1 - 16/x^2, x \geq 4$

Is continuous, if so find the associated probability density function ?

13. If X and Y are discrete random variables show that $E(X + Y) = E(X) + E(Y)$.

14. If $k(1 - e^{-2x})^3$ is a distribution function find the value of k .

15. What are the properties of a moment generating function ?

16. Write brief account on principle of least squares.

17. If X and Y are any two random variables, define $E(X/Y)$.

18. Differentiate between correlation and regression.

19. Distinguish between correlation and rank correlation.

20. Explain conditional expectation and show that $E(E(X/Y)) = E(X)$.

21. Show that when the correlation coefficient is zero, regression lines are perpendicular.

22. Derive normal equation in fitting a straight line.

(8x2=16 Marks)

SECTION - C

Answer **any six** questions, **each** question carries **4** marks :

23. X and Y are jointly discrete random variables with pmf given by $P(x, y) = \frac{1}{4}$ at $(x, y) = (-3, -5), (-1, -1), (1, 1), (3, 5)$. Compute $E(X)$, $E(Y)$, $E(X, Y)$. Are X and Y are independent.

24. A petrol pump is supplied with petrol once a day. If its daily volume of sales (X) in liters is distributed by : $f(x) = 5(1 - x)^4, 0 < x < 1$, what must be the capacity of its tank in order that probability of its supply will be exhausted in a given day shall be .01.



25. If X and Y are two independent random variables show that $V(X + Y) = V(X) + V(Y)$.
26. Two random variables X and Y have joint probability density function given by $f(x, y) = k(4 - x - y)$; $0 < x < 2$, $0 < y < 2$.
Find the value of k . Also find the marginal distributions of X and Y .
27. If X and Y are independent random variables show that $M_{(X+Y)}(t) = M_X(t)M_Y(t)$.
28. With usual notations show that $V(X) = E(V(X/Y)) + V(E(X/Y))$.
29. If $r(X, Y)$ is the correlation coefficient between X and Y show that $-1 \leq r(X, Y) \leq 1$.
30. Given two regression lines $2x - 3y = 5$, $5x - 8y = 9$; find means and correlation coefficient.
31. Find the correlation coefficient between X and Y .

X	1	2	3	4	5	6
Y	3	4	2	7	8	6

(6×4=24 Marks)

SECTION - D

Answer any 2 questions, each carries 15 marks :

32. The joint probability mass function of X and Y are as given below.

X \ Y	1	2	3	4
1	.1	.05	.02	.02
2	.05	.20	.05	.02
3	.02	.05	.20	.04
4	.02	.02	.04	.10

Find marginal distributions of X and Y . Find conditional distribution of X given $Y = 2$ and conditional distribution of Y given $X = 4$. Also find correlation between X and Y .



33. Let X and Y have the joint probability density function $f(x, y) = k(x - y)$, $0 \leq x \leq y \leq 1$ and zero elsewhere. Find k . Examine whether X and Y are independent. Also find covariance between X and Y .
34. Derive normal equations for fitting a second degree curve. Also fit the linear regression of Y on X for the following data.

X	2	4	6	8	10	12	14	16
Y	2.8	3.1	4.00	4.76	4.90	5.6	8.20	10.37

35. Derive formula for rank correlation coefficient.

Derive the rank correlation between X and Y for the following data.

X	2	7	3	6	4	5
Y	1	6	4	5	7	2

(2×15=30 Marks)
