



Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2019

First Degree Programme Under CBCSS

Mathematics

Core Course – XI

MM 1643 – COMPLEX ANALYSIS – II

(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are **compulsory**. Each carries **1** mark.

1. Write the power series expansion of $f(z) = \frac{1}{1+z}$ in the disc $|z| < 1$.
2. Test whether the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ is convergent in $\mathbb{C}$.
3. What are the singular points of $ze^{\frac{1}{z}}$?
4. Define essential singularity for a function $f(z)$ at $z = a$.
5. Describe the nature of singularity for $f(z) = \frac{1-e^{2z}}{z^4}$ at $z = 0$.
6. What is the order of the pole for $f(z) = \frac{1}{(2\sin z - 1)^2}$?
7. Find the residue of $f(z) = \frac{e^z}{z^2}$ at its singularity.
8. What is the residue of $f(z) = \frac{1}{z} + 1 + z + z^2 + \dots$ at $z = 0$?
9. What is the order of the zero of $z(e^z - 1)$?
10. If f is an even function, then what is the value of $\int_{-\infty}^{\infty} f(x) dx$?

P.T.O.



SECTION - II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Find a power series expansion for $f(z) = \frac{z-1}{z+1}$ about $z = 0$.
12. Prove that $\int_C \frac{\sin z}{\left(z - \frac{\pi}{2}\right)^2} dz$, where C is the circle $|z| = 2$.
13. Evaluate $\int_C \frac{e^{iz}}{z} dz$ where C is $|z| = 2$.
14. Determine and identify the singularities of $\frac{z}{1+z^2}$.
15. Determine the order of the pole and residue at $z = 0$ for $\frac{\sinh z}{z^4}$.
16. Find the residue of $\cot z$ at $z = 0$.
17. Write the principal part of the function $f(z) = z \exp\left(\frac{1}{z}\right)$ at its isolated singular point and determine the value of the singularity.
18. If a is a zero of order r for $\frac{1}{f(z)}$, then prove that a is a pole of order r for $f(z)$.
19. Find the residue of $\frac{1}{(z^2 + a^2)^2}$ at $z = ai$.
20. State Jordan's Lemma.
21. Find the residue of $\frac{ze^z}{(z-1)^3}$ at its pole.
22. Find $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

SECTION - III

Answer **any six** questions from this Section. **Each** question carries **4** marks.

23. State and prove Cauchy's theorem.
24. Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$.
25. Evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2} dz$ where C is $|z| = 3$.



26. Let f be analytic inside and on a simple closed curve C . Then prove that

$$f'(z) = \frac{1}{2\pi i} \int_C \frac{f(\xi)}{(\xi - z)^2} d\xi, \text{ where } z \text{ is any point inside } C.$$

27. Let f be a function which is bounded and analytic throughout a domain $0 < |z - z_0| < \delta$. Then prove that either f is analytic at z_0 or else z_0 is a removable singular point of δ .

28. Prove that an isolated singularity a of $f(z)$ is a pole if and only if $\lim_{z \rightarrow a} f(z) = \infty$.

29. Prove that $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{2\pi}{3}$.

30. Prove that $\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta} = \frac{\pi}{\sqrt{a^2 + 1}}$, $a > 0$.

31. Find $\sum_{n=0}^{\infty} \binom{2n}{n} \frac{1}{5^n}$.

SECTION - IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) State and prove Cauchy's integral formula. 7

b) Find the residue of $\frac{e^z}{z^2(z^2 + 9)}$ at its poles. 8

33. a) State and prove Casorati-Weierstrass theorem. 7

b) Prove that $\int_{-\infty}^{\infty} \frac{(x^2 - x + 2)}{x^4 + 10x^2 + 9} dx = \frac{5\pi}{12}$. 8

34. a) Prove that $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2} = \frac{\pi}{4a^3}$. 8

b) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. 7

35. a) Use residue, evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a \sin \theta} = \frac{2\pi}{\sqrt{1 - a^2}}$, $-1 < a < 1$. 8

b) Prove that $\int_0^{\infty} \frac{\cos x}{1 + x^2} dx = \frac{\pi}{2e}$. 7