



Reg. No. :

Name :

Second Semester B.A. Degree Examination, July 2017
First Degree Programme Under CBCSS
Complementary Course for Economics
MM 1231.5 : MATHEMATICS FOR ECONOMICS - II
(2013 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION - I

All the first ten questions are **compulsory**. They carry **1 mark each**.

1. Find the third derivative of $y = x^5$.
2. Determine whether the function $y = 3x^2 - 14x + 5$ is increasing or decreasing at $x = 4$.
3. What is the second derivative of $y = (2x + 3)(8x^2 - 6)$?
4. Find the value of x at which the function $y = 40 - 6x - x^2$ attains minimum value.
5. Find the marginal revenue function, if the total revenue function of a good is $100x - x^2$.
6. Find the degree of the homogeneous function $z = \frac{\sqrt{x^4 + xy^3}}{x + y}$.
7. Check whether $y = x^3 - x^2$ concave or convex on the interval $[-10, 0]$.
8. Write down the explicit form of the function $x^3 - xz + yz + z = 0$.
9. If $f(x, y) = \frac{1}{\sqrt{x^2 - y^2}}$, find $\frac{\partial f}{\partial y}$.
10. Find the point of inflexion of the curve $y = (x - 5)^3 + 3$.



SECTION - II

Answer any 8 questions from among the questions 11 to 22. These questions carry 2 marks each.

11. Find the maximum or minimum value of $f(x) = 5(x-3)^2 + 4$.
12. Check whether the function $y = x^3 + x + 10$ is monotonic or not.
13. If $y = ax^2 + bx$, show that $\frac{x^2}{2} \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$.
14. Find $\frac{dy}{dx}$ when $x = t^3 - t$ and $y = 4 - t^2$.
15. Find the maximum and minimum value of $y = 2x^3 - 9x^2 + 12x - 3$.
16. Find the point of inflexion of the curve $y = x^3 - 18x^2 + 96x - 80$.
17. Show that the demand curve $p = 2x - 3 + \frac{1}{x}$ is downward sloping and convex from below for all positive value of x .
18. Find $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ for $f(x, y) = x^2y + xy^2$.
19. Find dy if $y = \log(x^2 + 1)$.
20. Find du if $u = x^2 + xy$.
21. Verify Young's theorem for the function $z = e^{x-y}$.
22. Find the extreme points of $f(x, y) = 3x^2 - xy + 2y^2 - 4x - 7y + 12$.

SECTION - III

Answer any 6 questions from among the questions 23 to 31. These questions carry 4 marks each.

23. Obtain the extreme values of $f(x) = 3x^5 - 5x^2 + 1$.
24. Find the first and second order differentials of $z = 3x^2 + 12xy + 5y^2$.



25. Find $\frac{dy}{dx}$, when x and y are connected by the relation $x^2 y^2 = x^3 + y^3 - 3xy$.
26. An open box consists of a square base with vertical sides and has a volume of 4 cubic feet. What are the dimensions of the box for a minimum surface area.
27. If $xy = a$ show that $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0$.
28. If the cost function C of a product is given by $C = 2x \left(\frac{x+7}{x+5} \right) + 7$. Prove that the marginal cost falls continuously as the output increases.
29. If $u = x^3 - 3xy^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
30. Obtain the extreme values of $f(x, y) = x^3 + y^3 - xy + y^2 + 4$.
31. Find the minimum of $u = xy$ under the condition that $2x + y = 6$. What is the value of u ?

SECTION - IV

Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each**.

32. a) State Euler's theorem of homogeneous function and verify it for the function $f(x, y) = x^3 + 2x^2y + y^3$.
- b) The cost function for a firm is given by $C = 300x - 10x^2 + \frac{1}{3}x^3$, where x is the output.
Determine :
- i) the output at which marginal cost is minimum,
 - ii) the output at which average cost is minimum and
 - iii) the output at which average cost is equal to the marginal cost.



33. a) Given $u = x^2 + xy + y^2$, $x = t^3 + 1$, $y = t - t^3$ find $\frac{du}{dt}$ and $\frac{d^2u}{dt^2}$.
- b) If $z = \log(x^2 + y^2)$ show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.
34. a) The cost function for x units of a product produced and sold by a company is $C = 250 + 0.005x^2$ and the total revenue is given as $R = 4x$. Find how many items should be produced to maximize the profit. What is the maximum profit?
- b) If $f(x, y) = x \cos x + ye^x$, find the second order derivatives.
35. a) A company has established that the revenue function is $R(x) = 2x^3 + 40x^2 + 8x$ and the cost function is $C(x) = 3x^3 + 19x^2 + 80x - 800$. Find the price per unit to maximize the profit.
- b) If $y = x + \sqrt{x^2 - a^2}$, prove that $(x^2 - a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$.
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