

Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, March 2020

First Degree Programme under CBCSS

Mathematics

Core Course X — MM 1642 — LINEAR ALGEBRA

(2014 admission onwards)

Time : 3 Hours

Max. Marks : 80

SECTION - I

All the first ten questions are compulsory. Each carries 1 mark.

1. For which choice of x , will the following pair $\left(\begin{pmatrix} x \\ x^2 \end{pmatrix}, \begin{pmatrix} 1 \\ x \end{pmatrix}\right)$ be linearly dependent?
2. What is the equation of the line along the vector $\begin{pmatrix} -b \\ a \end{pmatrix}$?
3. Find the angle between $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.
4. Find the projection of $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ to the line along $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.
5. Find the distance from $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ to the line $y = 2x$.
6. Let P be the transformation which assigns to each vector X the projection of X on the line along the vector $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. If $X = \begin{pmatrix} x \\ y \end{pmatrix}$, find $P(X)$.

7. Let $X = \begin{pmatrix} x \\ y \end{pmatrix}$, calculate $R_\pi(X)$.
8. Let P be the linear transformation of the plane, given by projection to the line along $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Write down the matrix of the linear transformation.
9. What are the eigen values of the identity transformation on the plane.
10. Let R_θ and R_ϕ be rotations by angles θ radians and ϕ radians respectively. Describe geometrically $R_\theta R_\phi$.

SECTION – II

Answer **any eight** questions from this section. **Each** question carries **2** marks.

11. Write an equation of the line through $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ parallel to the vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$.
12. Find all solutions to the system of equations

$$\begin{aligned} 5x + y &= 0 \\ -10x - 2y &= 0 \end{aligned}$$
13. Calculate $R_{\frac{\pi}{4}}(X)$ if $X = \begin{pmatrix} x \\ y \end{pmatrix}$.
14. Check whether the transformation which send $\begin{pmatrix} x \\ y \end{pmatrix}$ into $\begin{pmatrix} x \\ x^2 + y \end{pmatrix}$ is a linear transformation.
15. Find a linear transformation which is equal to $R_{\frac{3\pi}{2}}$.
16. Let A be an arbitrary linear transformation. Prove that A sends the origin into the origin.
17. Let A the transformation with matrix $\begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$. Find the image of the line along $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ under A .

18. Let B be the reflection in the x -axis and A is the reflection in the y -axis. Find BA .
19. Let A be a linear transformation and let I denote the identity transformation. Prove that $AI = IA = A$.
20. Let H_k be the transformation with matrix $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$ show that $H_k H_m = H_{k+m}$.
21. Define elementary matrices with examples.
22. Construct a linear transformation with eigen values 3 and 4.

SECTION – III

Answer **any six** questions from this section. **Each** question carries **4** marks.

23. Let R_θ be the rotation transformation through angle θ radians counterclockwise. If $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $R_\theta(X) = \begin{pmatrix} x' \\ y' \end{pmatrix}$, prove that $\begin{aligned} x' &= (\cos \theta)x - (\sin \theta)y \\ y' &= (\sin \theta)x + (\cos \theta)y \end{aligned}$.
24. Prove that if A is a linear transformation in the plane, then $A(X + \bar{X}) = A(X) + A(\bar{X})$ for every pair of vectors $X, \bar{X}$.
25. Let P denote projection on the line along $u = \begin{pmatrix} u \\ v \end{pmatrix}$. Prove that $P(E_1) = \begin{bmatrix} \frac{u^2}{u^2 + v^2} \\ \frac{uv}{u^2 + v^2} \end{bmatrix}$ and $P(E_2) = \begin{bmatrix} \frac{uv}{u^2 + v^2} \\ \frac{v^2}{u^2 + v^2} \end{bmatrix}$ where $E_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $E_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. What is matrix of P ?
26. Write a linear transformation in $\mathbb{R}^4$ and find the corresponding matrix.
27. Prove that the commutative law of multiplication does not hold for the product of linear transformations.

28. Let U be the linear transformation having matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.
- (a) Interpret u geometrically
- (b) Show that $UU = I$.
29. Let L be a straight line through the origin and θ be the angle from the positive x -axis to L . Find the matrix of the transformation S which reflects each vector in L .
30. Let A be a linear transformation with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$. Prove that if π is any parallelogram with one vertex at 0, then $\text{area}(A(\pi)) = \left(\text{absolute value of } \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right) \text{area}(\pi)$.
31. Describe the curve $x^2 + 2xy + y^2 = 1$.

SECTION – IV

Answer **any two** questions from this section. **Each** question carries **15** marks.

32. Let A be a linear transformation and let L be a straight line. Prove that the image of L under A is either a straight line or a single point.
33. Prove that if A, B, C are three linear transformations, then $(AB)C = A(BC)$ and $(m(A)m(B)m(C)) = m(A(m(B)m(C)))$.
34. Let A be a linear transformation with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Prove that if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} > 0$, then A preserves orientation and if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} < 0$, then A does not preserve orientation.
35. Let A be a linear transformation with symmetric matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and let t_1, t_2 be distinct eigenvalues of A . Choose nonzero eigenvectors X_1, X_2 corresponding to t_1 and t_2 respectively. Prove that X_1 and X_2 are orthogonal.