

29. Row reduce the matrix $\begin{pmatrix} 1 & 2 & -3 & -4 \\ 1 & 3 & 1 & -2 \\ 2 & 5 & -2 & -5 \end{pmatrix}$.

30. Solve $(D^2 - 3D + 2)y = \sin 3x$.

31. Define Bernoulli's and Legendre's equation with examples.

SECTION - IV

(6 × 4 = 24 Marks)

Answer any two questions. These questions carry 15 marks each.

32. Verify Stokes' theorem when $F = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ around S where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

33. Solve $(2x - 3y + 1)dx + (6y - 4x + 3)dy = 0$.

34. Diagonalize the matrix $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

35. Solve by using the method of variation of parameter $\frac{d^2y}{dx^2} + a^2y = \cos ax$.

(2 × 15 = 30 Marks)

(Pages : 4)

S - 3420

Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III - CALCULUS AND LINEAR ALGEBRA

(2018-2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION - I

All the questions are compulsory. They carry 1 mark each.

- Find the gradient of the function $F(x, y, z) = 7x^2 + 9y^2 + 7xyz$.
- Prove that for any scalar function $F(x, y, z)$ whose second partial derivatives are continuous, prove that $\text{curl}(\text{grad}F) = 0$.
- Find the general solution of $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$.
- Form the differential equation of the family of circles touching the x axis at the origin.
- Find the Wronskian of $\sin x$, $\sin 3x$.
- Find the general solution of the differential equation $ydx - (x + 2y^2)dy = 0$.

7. Find the rank of $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 8 & 12 \\ 2 & 4 & 6 \end{pmatrix}$.

8. Find the eigen values of $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 9 \end{pmatrix}$.

9. Evaluate the determinant of $\begin{pmatrix} 5 & 6 & 7 \\ -2 & -1 & 0 \\ 1 & 2 & -3 \end{pmatrix}$.

10. Find the average value of $f(x) = x^3$ on $(-1, 2)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any eight questions. These question carries 2 marks each.

11. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\text{div } \vec{r} = 3$ and $\text{curl } \vec{r} = 0$.

12. What is the matrix corresponding to the quadratic form $x^2 + 2y^2 - 2xy + 3xz - 2yz + z^2$?

13. Use double integral to find the volume of the tetrahedron bounded by the coordinate planes $z = 4 - 4x - 2y$.

14. Determine whether $ax^2 + by^2 = 1$ is a solution of $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$.

15. Is $(x^2 - y^2)dx + 2xydy = 0$ is a homogeneous differential equation? Solve this differential equation.

16. Solve $(e^y + 1)\cos x dx + e^y \sin x dy = 0$.

17. Solve $(y^2 + 1) + (x - e^{-\tan^{-1}y}) \frac{dy}{dx} = 0$.

18. What is the complementary function of $(D^3 + 3D^2 - 4)y = xe^{-2x}$.

19. Are the vectors $(3, -2, 8)$, $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$ linearly independent.

20. If λ is a nonzero eigenvalue of a nonsingular matrix A, then show that λ^{-1} is an eigen value of A^{-1} .

21. Find the complimentary function of $(D^2 + a^2)y = \text{Cot } ax$.

22. Represent the following equations in the matrix form $y + z - 2w = 0$

$$2x - 3y - 3z + 6w = 2$$

$$4x + y + z - 2w = 4$$

(8 × 2 = 16 Marks)

SECTION – III

Answer any six questions. These questions carry 4 marks each.

23. Find the fourier series of the function $f(x)$ of period 4 where $f(x) = \begin{cases} 0, & -2 \leq x \leq 0 \\ 1, & 0 < x < 2 \end{cases}$

24. Find the area enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$.

25. Show that the integral of $\mathcal{F} = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$ is path independent. Also find the integral over C from $(0, 0, 0)$ to $(2, 2, 2)$.

26. Show that $F = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative and find a potential function for it.

27. Evaluate $\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the xy -plane bounded by the x -axis, the line $y = x$ and the line $x = 1$.

28. Integrate $f(x, y, z) = x + \sqrt{y} - z^2$ over the path from $(0, 0, 0)$ to $(1, 1, 1)$ given by $C_1: r(t) = t\hat{i} - t^2\hat{j}, 0 \leq t \leq 1$.