

30. Prove that eigen vectors corresponding to different eigen values are linearly independent.

31. Find the eigen values of the matrix $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. Solve the system of linear equations

(a) $x + y + 2z = 8$

$-x - 2y + 3z = 1$

$x - 7y + 4z = 10$

(b) $2u + v + w = 5$

$4u - 6v = -2$

$-2u + 7v + 2w = 9$

33. If $v_1, \dots, v_m$ and $w_1, \dots, w_n$ are both bases for the same vector space, then prove that $m = n$.

34. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

35. (a) Prove that diagonalizable matrices share the same eigen vector matrix S if and only if $AB = BA$.

- (b) Suppose the n by n matrix A has n linearly independent eigen vectors. If these eigen vectors are the columns of a matrix s , then prove that $S^{-1}AS$ is a diagonal matrix A .

(2 × 15 = 30 Marks)

(Pages : 4)

T – 1609

Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : LINEAR ALGEBRA

(2018-2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find the inner product of $\begin{bmatrix} 1 & 2 & 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

2. When is an upper triangular matrix nonsingular?

3. Define column space of a matrix.

4. Give an example of a matrix of rank 3.

5. Define a symmetric matrix.

6. What is the dimension of the space of all 2×2 diagonal matrices?

7. Explain cofactor matrix.

8. How is the determinant of a matrix related to its eigen values?
9. Give an example of a matrix that is not diagonalizable.
10. How does the trace of a matrix change when the matrix is transposed?

(10 × 1 = 10 Marks)

SECTION – II

Answer any eight questions. These questions carry 2 marks each.

11. 'If A and B are $n \times n$ matrices, then $(A+B)^2 = A^2 + 2AB + B^2$. State whether the statement is true? Justify your answer.
12. Explain permutation matrix with an example.
13. Explain triangular factorization of a matrix A .
14. Show that the set of lower triangular matrices is a sub space of set of all $n \times n$ matrices.
15. Check whether the set $\{(x, y) / x + 2y = 1\}$ is a subspace of $\mathbb{R}^2$.

16. Reduce the matrix into echelon form $\begin{bmatrix} 0 & 1 & 5 & 2 \\ 3 & -6 & 9 & 7 \\ 2 & 6 & 1 & 4 \end{bmatrix}$.

17. Find rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 2 & 8 & 10 \end{bmatrix}$.

18. Check whether $\left\{ (1, 0, -1), \left(\frac{-1}{2}, 0, \frac{1}{2} \right), (4, 1, 1) \right\}$ is a basis of $\mathbb{R}^3$.

19. Check whether the transformation $T(x, y) = (x, y + 1)$ is linear or not?

20. Give an example of a linear transformation from $\mathbb{R}$ to $\mathbb{R}^2$ and justify your claim.
21. What is the determinant of a matrix if one of its rows is a linear combination of the other rows? Give reasons.
22. Explain Hermitian matrix with an example.

(8 × 2 = 16 Marks)

SECTION – III

Answer any six questions. These questions carry 4 marks each.

23. Using Gauss-Jordan method, compute the inverse of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$$

24. Which of the following subsets of $\mathbb{R}^3$ are subspaces?

- (a) The set of all vectors of the form $(x, 0, 0)$.
- (b) The set of all vectors the form $(x, 1, 1)$.

25. Determine whether the vectors $u_1 = (1, 1, 0)$, $u_2 = (1, -1, 3)$, $u_3 = (4, -2, 9)$ spans $\mathbb{R}^3$.
26. Show that the vectors $(3, 5, -2, 6)$, $(2, 4, 1, 5)$ and $(1, 2, -1, 1)$ are linearly independent.
27. For square matrices A and B , prove that $|AB| = |A| |B|$.
28. Prove that row rank = column rank.
29. Solve by Cramer's rule.

$$\begin{aligned} x + 3y &= 0 \\ 2x + 4y &= 6 \end{aligned}$$