

PART – D

Answer any **two** questions. These question carry **15** marks each.

32. (a) Let A be the set of integers, and let $\sim$ be the relation on $A \times A$ defined by $(a, b) \sim (c, d)$ if $a + d = b + c$;
- (i) Prove that $\sim$ is an equivalence relation.
- (ii) Suppose $A = \{1, 2, 3, \dots, 8, 9\}$, find the equivalence class of $(2, 5)$.
- (b) Find all partitions of $S = \{a, b, c, d\}$
33. (a) Give a proof of the theorem "If $3n + 2$ is odd, then n is odd" by the method of contraposition.
- (b) Find the area inside the circle $r = 3 \sin \theta$ and outside the cardioid $r = 1 + \sin \theta$.
34. (a) Describe and draw the graph of the equation $16x^2 + 9y^2 - 64x - 54y + 1 = 0$.
- (b) Describe and draw the graph of the equation $x^2 - y^2 - 4x + 8y - 21 = 0$.
- (c) State Kepler's laws.
35. (a) Find the distance D between the point $(1, -2, 3)$ and the plane $2x - 2y + z = 4$.
- (b) Sketch the graph of the hyperboloid of one sheet $x^2 + y^2 - \frac{z^2}{4} = 1$.
- (c) Find the rectangular coordinates of the point with cylindrical coordinates $(r, \theta, z) = (2, \frac{\pi}{4}, -5)$.

(2 × 15 = 30 Marks)

(Pages : 4)

R – 3102

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2023

First Degree Programme under CBCSS

Mathematics

Foundation Course – II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

PART – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define Universal quantifier.
2. Find the cartesian product of $A = \{a, b\}$ and $B = \{y, z\}$.
3. Examine whether the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is one-one.
4. How many relations are there on a set with n elements?
5. Draw the graph of the parametric equations $x = \cos t$, $y = \sin t$ where $0 \leq t \leq 2\pi$.
6. Define a parabola.
7. Find the eccentricity and distance from the pole to the directrix for the curve $r = \frac{3}{2 - 2\cos \theta}$.
8. Find the distance between the points $(2, 7, 6)$ and $(5, 4, -2)$.

9. Find the norm of the vector $v = (2, 3, 6)$.
10. Let $u = (1, 2, -2)$ and $v = (3, 0, 1)$. Find $v \times u$.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. These questions carry **2** marks each.

11. Show that $(p \wedge q) \rightarrow p$ is a tautology by using truth table.
12. Is ' $x + 2 = 12$ ' a statement? If not rewrite it as statement.
13. Let $Q(x)$ be the statement " $x < 2$ ". What is the truth value of the quantification for every $x, Q(x)$, where the universe of discourse is the set of real numbers?
14. Find the domain D of the following real valued functions.
- (a) $f(x) = \frac{1}{x-2}$
- (b) $g(x) = x^2 - 3x - 4$
15. Consider the following relation R on $S = \{1, 2, 3, 4\}$, $R = \{(1, 1), (2, 2), (1, 3), (3, 1), (3, 3), (4, 4)\}$. Show that R is an equivalence relation. Also find the equivalent classes.
16. Find the arc length of the curve $x = \cos 3t, y = \sin 3t$ over the interval $0 \leq t \leq \pi$.
17. Express the relation between the polar and the rectangular coordinates.
18. Check whether the graph of $r = \cos 2\theta$, symmetric about the x-axis and y-axis.
19. Find the entire area of the cardioid $r = 1 + \cos \theta$.
20. Find a vector that is orthogonal to both the vectors $u = (1, 2, -3)$ and $v = (5, 2, -6)$.
21. Find the parametric equations of the line passing through $(1, 3)$ and parallel to $v = (2, -5)$.
22. Determine whether the planes $2x - y + 7z = 0$ and $-4x + 2y - 14z - 5 = 0$ are parallel.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. These questions carry **4** marks each.

23. Prove that $\sqrt{2}$ is irrational by giving a proof by the method of contradiction.
24. Construct a truth table for the following compound proposition $(p \rightarrow q) \vee (\neg p \rightarrow r)$.
25. Consider the functions $f : A \rightarrow B$ and $g : B \rightarrow C$. Prove the following :
- (a) If f and g are one-to-one functions, then the composition function $g \circ f$ is one-to-one.
- (b) If f and g are onto functions, then the composition function $g \circ f$ is an onto function.
26. Find the slope of the tangent line to the circle $r = 4 \cos \theta$ at the point where $\theta = \frac{\pi}{4}$.
27. Find the total arc length of the cardioid $r = 1 - \cos \theta$.
28. Sketch the graph of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ and find their vertices, foci and asymptotes.
29. Show that $(2, 1, 6)$, $(4, 7, 9)$ and $(8, 5, -6)$ are the vertices of a right triangle. Find the vertex which is at the 90° angle. Find the area of the triangle.
30. Find the area of the triangle that is determined by the points $P_1(2, 2, 0)$, $P_2(-1, 0, 2)$ and $P_3(0, 4, 3)$.
31. (a) Find the vector of length 3 that makes an angle of $\frac{\pi}{6}$ with the positive x-axis.
- (b) Find the angle that the vector $v = i + j$ makes with the positive x-axis.

(6 × 4 = 24 Marks)