



Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, December 2017
(First Degree Programme Under CBCSS)

MATHEMATICS

Core Course - 2

MM 1341 : Algebra and Calculus - 1
(2014 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

All the first 10 questions are compulsory. They carry 1 mark each.

1. Is the set $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ a ring with respect to addition modulo 4 and multiplication modulo 4?
2. Give an example of a ring which has no zero divisors.
3. Find $[2]_5 \cdot [3]_5$.
4. Define exponent of a group.
5. Find the distance from the point $(-7, 5, 2)$ to the xy -plane.
6. Fill in the blanks :

In 3-space, the equation $x^2 + z^2 = 10$ represents a _____.

7. Find the cosine of the angle between the vectors $\vec{u} = \langle 0, 2, 3 \rangle$ and $\vec{v} = \langle 0, -2, 1 \rangle$.

8. Give the natural domain for the vector valued function.

$$\vec{r}(t) = (\ln |t-6|) \vec{i} + 7t^2 e^t \vec{j} + \sqrt{t-2} \vec{k}$$

9. Find the slope of the line in 2 space that is represented by the vector equation

$$\vec{r} = (12-7t) \vec{i} - (5-4t) \vec{j}.$$

10. State Kepler's Law of Areas.

(10×1=10 Marks)

P.T.O.



Answer **any 8** questions from among the questions **11 to 22**. These questions carry **2 marks each**.

11. Justify that the set $\mathbb{Z} - \{3\}$ of all integers except 3 is not a ring with respect to the usual addition and multiplication.

12. Prove that in $\mathbb{Z}/m\mathbb{Z}$, $[a]_m$ is a unit if and only if $(a, m) = 1$.

13. If R and S are rings and the function $f: R \rightarrow S$ is a homomorphism, then prove that $f(-r) = -f(r)$ for any r in R .

14. Find the unit vector in the same direction from the point $A(-3, 2)$ to the point $B(1, -1)$.

15. If $\vec{u}$ and $\vec{v}$ are vectors in 3-space, then prove that $\vec{v} \cdot (\vec{u} \times \vec{v}) = 0$.

16. Find $\vec{u} \cdot (\vec{v} \times \vec{w})$ where $\vec{u} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{v} = 4\vec{i} + \vec{j} - 3\vec{k}$, $\vec{w} = \vec{j} + 5\vec{k}$.

17. Find parametric equations for the line through $(-5, 2)$ that is parallel to $2\vec{i} - 3\vec{j}$.

18. Find a spherical coordinate equation of the Cartesian equation $z = \sqrt{x^2 + y^2}$.

19. Convert the following spherical coordinates to rectangular coordinates:

a) $(7, 0, \pi/2)$

b) $(1, \pi, 0)$

20. If $\vec{r}(t)$ is a vector-valued function in two or three dimensional space and $\|\vec{r}(t)\|$ is constant for all t , then prove that $\vec{r}(t)$ and $\vec{r}'(t)$ are orthogonal vectors for all t .

21. Find a change of parameter $t = g(\tau)$ for the semicircle

$$\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j}, \quad 0 \leq t \leq \pi$$

such that the semicircle is traced counter clockwise as τ increases over the interval $[0, 1]$.

22. Find the curvature for the helix

$$\vec{r}(t) = (a \cos t) \vec{i} + (a \sin t) \vec{j} + bt \vec{k} \quad a, b \geq 0, a^2 + b^2 \neq 0.$$

(8x2=16 Marks)



Answer any 6 questions from the questions 23 to 31. These questions carry 4 marks each.

23. Let R be a commutative ring with at least 5 elements. Prove that the equation $x^2 - rx + s = 0$ has at most two solutions in R for every r, s in R , if and only if R has no zero divisors.

24. Define field. Give one example.

25. Find the coordinates of the centre and the radius of the sphere $x^2 + y^2 + z^2 - 4x + 2y - 2z - 10 = 0$.

26. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vectors in 3-space, then prove that

$$\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w}).$$

27. Determine whether the following lines are parallel, intersect or are skew. If they intersect, find the point of intersection.

(Line 1) : $x = 3 + 2t, y = 2 + t, z = -2 + 2t, -\infty < t < \infty$

(Line 2) : $x = 1 + 4s, y = 1 + 2s, z = -3 + 4s, -\infty < s < \infty$

28. Determine whether the following vector-valued function has continuously turning tangent vector :

$$\vec{r}(t) = 3t^2 \vec{i} + (t^2 - 2t) \vec{j} + t^3 \vec{k}$$

29. Find the unit tangent vector at a point t to the curve $\vec{r} = a \cos t \vec{i} + a \sin t \vec{j}$.

30. Suppose that a particle moves through 3-space so that its position vector at time t is $\vec{r}(t) = t \vec{i} + t^2 \vec{j} + t^3 \vec{k}$.

Find

- the scalar tangential and normal components of acceleration at time $t = 1$.
- the curvature of the path at the point where the particle is located at $t = 1$.

31. A shell is fired from ground level with a muzzle speed of 320 ft/s and elevation angle of 60° . Find the horizontal distance travelled by the shell. (6×4=24 Marks)



Answer **any 2** questions from among the questions **32 to 35**. These questions carry **15 marks each**.

32. a) If R is any commutative ring, prove that the set of n -element column vectors for fixed n is an abelian group under addition.
 b) Let g_0 be the exponent of a finite abelian group G . Then the order of any element b of G divides g_0 .

33. a) State and prove the Chinese Remainder Theorem.

- b) Find a solution of the following simultaneous congruences :

$$x \equiv 3 \pmod{11},$$

$$x \equiv 6 \pmod{8},$$

$$x \equiv -1 \pmod{15}$$

Also find the smallest non-negative solution.

34. a) Find the point where the line

$$x = \frac{8}{3} + 2t, y = -2t, z = 1 + t$$

intersects the plane $3x + 2y + 6z = 6$.

- b) Identify the surface $x^2 - y + z^2 = 0$.

35. a) Find the unit tangent vector to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at the point $t = 1$.

- b) Suppose that a particle moves along a curve so that its position vector at

$$\text{time } t \text{ is } \left(\vec{r} = e^t \vec{i} + e^{-t} \vec{j} + \sqrt{2}t \vec{k} \right).$$

Find the distance traveled and the displacement of the particle during the time interval $0 \leq t \leq \ln 3$.

(2×15=30 Marks)