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S – 1631

Reg. No. :

Name :

Fifth Semester B.Sc. Degree Examination, December 2023

First Degree Programme under CBCSS

Mathematics

Core Course VI

MM 1542 : COMPLEX ANALYSIS I

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions. Theory carry **1** mark each.

1. Write the number $\frac{3}{i} + \frac{i}{3}$ in the form $a + bi$.
2. Write $-\pi i$ in polar form.
3. Find $\text{Arg}(-\pi)$.
4. State triangle inequality for two complex numbers z_1, z_2 .
5. Define entire function.
6. State Fundamental theorem of Algebra.
7. Define a branch of a multiple valued complex function $f(z)$ in a domain D .
8. Find a parametrization of the straightline segment from $z = i$ to $z = 3i$.

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9. State Jordan curve theorem.
10. Let C be a smooth curve given by parametrization $z(t) : a \leq t \leq b$. Give the length of this curve.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** question. These **Each** question carries **2** marks.

11. Show that $\operatorname{Re}(iz) = -\operatorname{Im}z$ for every complex number z .
12. Sketch $0 < |z - 2| < 3$. Is it a domain?
13. State True or False : $\operatorname{Arg}(z_1 z_2) = \operatorname{Arg}z_1 + \operatorname{Arg}z_2$. Justify your answer.
14. Prove that $|z_1 - z_2| \leq |z_1| + |z_2|$.
15. Find points at which $f(z) = z^2(2z^2 - 3z + 1)^{-2}$ is not analytic.
16. Find the points at which $f(z) = \frac{\sin^2 z}{z}$ is not analytic.
17. Prove that e^z is periodic with a period $2\pi i$.
18. Find $\operatorname{Log}(\sqrt{3} + i)$.
19. Finding an anti-derivative of e^{it} , evaluate $\int_0^\pi e^{it} dt$.
20. State True or False : $\int_{|z|=1} \bar{z} dz = \int_{|z|=1/2} \frac{1}{z} dz$. Justify your answer.
21. State Cauchy's integral theorem. Evaluate $\int_{|z|=2} f(z) dz = 0$, where $f(z) = \frac{z}{z^2 + 25}$.
22. State Deformation Invariance theorem.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. **Each** question carries **4** marks.

23. Describe the set of points that satisfy the equation $|z + 2| = |z - 1|$.
24. Using complex product $(1 + i)(5 - i)^4$, derive $\frac{\pi}{4} = 4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$.
25. Use De Moivre's formula and binomial formula to derive the identity $\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$.
26. Prove that if $f(z)$ is analytic in a domain D and if $f'(z) = 0$ in D , then $f(z)$ is constant in D .
27. Find all poles and their multiplicities for $f(z) = \frac{(3z + 3i)(z^2 - 4)}{(z - 2)(z^2 + 1)^2}$.
28. Find all values of i^i .
29. Determine the domain of analyticity of the function $f(z) := \text{Log}(3z - i)$. Compute $f'(z)$.
30. If C is the circle $|z| = 3$ traversed once, then show that $\left| \int_C \frac{dz}{z^2 - i} \right| \leq \frac{3\pi}{4}$.
31. State and prove Liouville's theorem.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Evaluate $\int_0^{2\pi} \cos^4 \theta d\theta$.
- (b) Verify $u = xy - x + y$ is harmonic and find a harmonic conjugate of u .

33. (a) Define $\sin z$. Prove that $\sin z$ is periodic with a period 2π .

(b) State and prove the necessary conditions (Cauchy-Riemann equations) for a complex function to be differentiable at a point.

34. Compute $\int_{\Gamma} \bar{z}^2 dz$ along the simple closed contour Γ , which is triangle with vertices $0, 2$ and $2 + i$ traversed in anti-clockwise direction.

35. Compute $\int_C \frac{z+i}{z^3 + 2z^2} dz$, where C is

(a) the circle $|z| = 1$ traversed once counterclockwise.

(b) the circle $|z + 2 - i| = 2$ traversed once counterclockwise.

(c) the circle $|z - 2i| = 1$ traversed once counterclockwise.

(2 × 15 = 30 Marks)