

Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : LINEAR ALGEBRA

(2018 Admission onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Write the given system of equations in column form.
 $5x + 20y = 80$
 $-x + 4y = -64$
2. Find two points on the line of intersection of the three planes $t=0, z=0$ and $x+y+z+t=1$ in four dimensional space.
3. Define a symmetric matrix.
4. Define subspace of a vector space.
5. The columns of a matrix A and n vectors from R^m . If they are linearly independent, what is the rank of A ?
6. Write the reflexion matrix that transforms (x,y) to (y,x) .
7. If a 3 by 3 matrix has $\det A = \frac{-1}{2}$, find $\det(A^{-1})$.

8. State whether true or false: The determinant of $S^{-1}AS$ equals the determinant of A .
9. If $\lambda \neq 0$ is an eigen value of A and x is its eigen vector, then show that x is also an eigen vector of A^{-1} .
10. Define a transition matrix.

(10 × 1 = 10 Marks)

SECTION – B.

Answer any **eight** questions. **Each** question carries **2** marks.

11. Write down the upper triangular system of equations. Also find the pivots
 $2x + 3y = 1$
 $10x + 9y = 11$
12. Give 2 by 2 matrices A and B such that $AB = 0$, although no entries of A or B are zero.
13. Describe the null space of $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.
14. Describe the subspace of $\mathbb{R}^3$ spanned by the two vectors $(1, 1, -1)$ and $(-1, -1, 1)$.
15. Check whether the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(V_1, V_2, V_3) = (V_2, V_3, 0)$ is linear.
16. Using the big formula, compute $\det A$ from 6 terms. Are rows independent?
 $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{bmatrix}$
17. The corners of a triangle are $(2, 1)$, $(3, 4)$ and $(0, 5)$. What is the area?
18. Suppose the permutation P takes $(1, 2, 3, 4, 5)$ to $(5, 4, 1, 2, 3)$. What does P^{-1} do to $(1, 2, 3, 4, 5)$.
19. Show that the eigen value of A equals the eigen value of A^T .
20. Factor $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ into $S\Lambda S^{-1}$.

21. Compute A^3 for the Fibonacci matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.
22. Prove that two eigen vectors of a Hermitian matrix, if they come from different eigen values are orthogonal to one another.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. For which numbers 'a' does the elimination break down
 $ax + 3y = -3$
 (i) Permanently (ii) Temporarily $4x + 6y = 6$

Solve for x and y after fixing the second break down by a row exchange.

24. Find all matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that satisfy $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} A$.

25. Reduce to echelon form $\begin{bmatrix} 0 & 0 & 2 \\ 1 & -1 & 1 \\ -1 & 1 & -4 \end{bmatrix}$.

26. Find the range and kernel of $T(V_1, V_2) = (V_1 V_1 + V_2)$.

27. Find x, y and z by Cramer's rule:

$$2x + y = 1$$

$$x + 2y + z = 70$$

$$y + 2z = 0$$

28. Define orthogonal matrix. Prove that if A is an orthogonal matrix, then $\det A = \pm 1$.

29. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.

30. If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find A^{100} by diagonalising it.

31. Compute $A^H A$ and AA^H for $A = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Compute the symmetric LDL^T factorisation of $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}$.

(b) Use the Gauss-Jordan method to invert $\begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & -4 \\ 2 & 3 & -1 \end{bmatrix}$.

33. (a) Find the value of α that makes the system solvable and find the solution

$$\begin{aligned} x + y + z &= 1 \\ 2x - y + 2z &= 1 \\ x + 2y + z &= \alpha \end{aligned}$$

(b) Find the dimensions of the column space and row space of $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

34. (a) Find the dimension and a basis for the four fundamental subspaces for

$$u = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Find the 4 by 4 matrix A that represents a right shift : (x_1, x_2, x_3) is transformed to $(0, x_1, x_2, x_3)$. Also find the left shift matrix B from R^4 back to R^3 ; transforming (x_1, x_2, x_3, x_4) to (x_2, x_3, x_4) .

35. (a) Test the Cayley-Hamilton theorem on $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

(b) Find a diagonal matrix M so that $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is similar to $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(2 × 15 = 30 Marks)