

Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, March 2021**First Degree Programme under CBCSS****Mathematics****Core Course XII****MM 1644 : LINEAR ALGEBRA****(2018 Admission Regular)**

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer all the first ten questions. Each carries 1 mark :

1. Find a point with $z=2$ on the intersection line of the planes $x+y+3z=6$ and $x-y+z=4$.
2. Find the inverses of $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$.
3. Give examples of A and B such that $A+B$ is not invertible although A and B are invertible.
4. Define Null Space of a matrix A .
5. Show that the vectors $(1, 0)$ and $(0, 1)$ are linearly independent.

6. True or false? "The determinant of $S^{-1}AS$ equals the determinant of A ".
7. Write Cramer's Rule

8. Find the sum of eigen values of $\begin{bmatrix} 3 & 2 & 5 & 7 \\ 7 & 1 & 4 & 9 \\ 6 & 9 & 11 & 23 \\ 1 & 3 & 77 & -5 \end{bmatrix}$.

9. Is the matrix $\begin{bmatrix} 3 & 3 & 15 \\ 0 & 4 & 8 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable and why?

10. Find the inner product of $\begin{bmatrix} 1+i \\ 2-3i \end{bmatrix}$ and $\begin{bmatrix} 1-i \\ 2+3i \end{bmatrix}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions among the questions 11 to 26. They carry **2** marks each.

11. Find the equation of a line that meets $x+4y=7$ at $x=3, y=1$.

12. The matrix $A(\theta)$ that rotates the $x-y$ plane by an angle θ is $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$.
Show that $A(\theta_1)A(\theta_2)=A(\theta_1+\theta_2)$.

13. For which three numbers c is this matrix not invertible, and why not?

$$\begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

14. Check whether $S = \{[abc] \in R^3 | a=b+1\}$ is a subspace of R^3 .
15. Show that the set of all solutions of the set of linear equations $Ax=b, b \neq 0$, under standard matrix addition and scalar multiplication is not a vector space.
16. Decide the dependence or independence of the vectors $(1,3,2), (2,1,3)$, and $(3,2,1)$.
17. Determine whether the transformation T is linear if $T:R^2 \mapsto R^1$ is defined by $T[a \ b] = ab$ for all real numbers a and b .
18. Let $S:M_{n \times n} \mapsto R$ map an $n \times n$ matrix into the sum of its diagonal elements. Such a transformation is known as the trace. Is it linear?
19. The corners of a triangle are $(2,1), (3,4)$, and $(0,5)$. What is the area?
20. Solve $3u+2v=7, 4u+3v=11$ by Cramers rule.
21. Suppose (x, y, z) is a linear combination of $(2,3,1)$ and $(1,2,3)$. What determinant is zero? What equation does this give for the plane of all combinations?
22. Suppose that λ is an eigenvalue of A , and x is its eigenvector: $Ax=\lambda x$. Show that this same x is an eigenvector of $B=A-7I$, and find the eigenvalue.
23. If a 3 by 3 upper triangular matrix has diagonal entries 1, 2, 7, how do you know it can be diagonalized? What is A ?
24. If B has eigenvalues 1, 2, 3. C has eigen values 4, 5, 6, and D has eigenvalues 7, 8, 9, what are the eigen values of the 6 by 6 matrix

$$A = \begin{bmatrix} B & D \\ 0 & C \end{bmatrix}.$$

25. Prove that "If $A=A^H$, every eigenvalue is real".
26. If $A+iB$ is a Hermitian matrix (A and B are real), show that $\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$ is symmetric.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions among the questions 27 to 38. They carry **4** marks each.

27. Solve by Gauss Elimination Method

$$x + y + z = 3$$

$$x + 2y + 3z = 0$$

$$x + 3y + 2z = 3$$

28. What three elimination matrices E_{21} , E_{31} , and E_{32} put A into upper-triangular form $E_{21}E_{31}E_{32}A = U$. Using these, compute the matrix L (and U) to factor $A = LU$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{bmatrix}$$

29. Determine whether the set of column matrices in R^3 $\left\{ \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 8 \\ 16 \\ -3 \end{bmatrix} \right\}$ linearly independent.

30. By applying row operations to produce an upper triangular U , compute

$$\det \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix}$$

31. Find a basis for the row space of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & -1 & 1 \\ 3 & 2 & 5 \\ 5 & 15 & 20 \end{bmatrix}$

32. A linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ has the property that

$$T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \text{ and } T \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Determine Tv for any vector $v \in \mathbb{R}^2$.

33. Use row operations to verify that the 3 by 3 "Vandermonde determinant" is

$$\det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (b-a)(c-b)(c-a).$$

34. Evaluate this determinant by cofactors of row 1:

$$\begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

35. Find x , y , and z by Cramers Rule in equation

$$x + 4y - z = 1$$

$$x + y + z = 0$$

$$2x + 3z = 0$$

36. Diagonalize the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$.

37. Prove that "Diagonalizable matrices share the same eigenvector matrix S if and only if $AB = BA$ ".

38. Write the matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ in the form $U^{-1}AU = T$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions among the questions 39 to 44. They carry **15** marks.

39. (a) Which number q makes this system singular and which right-hand side t gives it infinitely many solutions? Find the solution that has $z = 1$.

$$x + 4y - 2z = 1$$

$$x + 7y - 6z = 6$$

$$3y + qz = t$$

- (b) For the system

$$u + v + w = 2$$

$$u + 3v + 3w = 0$$

$$u + 3v + 5w = 2$$

what is the triangular system after forward elimination, and what is the solution?

40. (a) Using the Gauss-Jordan Method to Find A^{-1}

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}.$$

- (b) Show that the set of all even functions defined on R , that is $f(x) = f(-x)$ is a vector space.

41. (a) Determine whether the transformation R is linear, if R is defined by

$$R \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \cos \theta & -b \sin \theta \\ a \sin \theta & +b \cos \theta \end{bmatrix}$$

where a and b denote arbitrary real numbers and θ is a constant.

- (b) Find a basis for each of these subspaces of 3 by 3 matrices:

- (i) All diagonal matrices.
- (ii) All symmetric matrices.

42. (a) By applying row operations to produce an upper triangular U , compute

$$\det \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix}.$$

- (b) Suppose the permutation P takes $(1, 2, 3, 4, 5)$ to $(5, 4, 1, 2, 3)$

(i) What does P^2 do to $(1, 2, 3, 4, 5)$?

(ii) What does P^{-1} do to $(1, 2, 3, 4, 5)$?

43. Let $A = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

(a) Find all eigen values of A .

(b) Find a maximum set of linearly independent eigen vectors of A

(c) Is A diagonalizable? If yes, find S such that $A = S \Lambda S^{-1}$.

44. (a) B is similar to A and C is similar to B , show that C is similar to A

(b) Explain why A is never similar to $A + I$

(c) Show (if B is invertible) that BA is similar to AB .

(2 × 15 = 30 Marks)
