

Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS I

(2018 Admission onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the questions are compulsory. **Each** question carries **1** mark

State the Pigeonhole Principle.

State the Fundamental Theorem for Arithmetic.

Define continuity of a vector valued function.

If $r_0 = r(t_0)$, $v_0 = r'(t_0)$, give the vector equation of the tangent line to the graph of $r(t)$ at r_0 .

Define Unit Tangent Vector.

Give the formula for a_T , the tangential component of acceleration for a moving particle in terms of the velocity v and the acceleration a .

P.T.O.

7. Find the natural domain of $f(x, y) = \sqrt{y+1} + \ln(x^2 - y)$.
8. Define boundary point of a set.
9. Define directional derivative of $f(x, y, z)$ in the direction u .
10. Give the equation of the tangent plane to a level surface S at a point (x_0, y_0, z_0) .

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. **Each** question carries **2** marks

11. Prove that there are infinitely many primes.
12. Express 3014 in base 8.
13. Prove that if p is a prime and $p \nmid ab$, then $p \nmid a$ or $p \nmid b$.
14. Give the general solution of an LDE $ax + by = c$, if (x_0, y_0) is its particular solution.
15. Find the parametric equations that represent a line in 3-space that passes through the point $(1, 0, 0)$ and is parallel to the vector $(-1, 3, 2)$.
16. Find the derivative of $r(t) = t^2 i + e^t j - (2 \cos \pi t) k$.
17. If $r_1(t)$ and $r_2(t)$ are two vector functions of t , derive the expression for $\frac{d}{dt}(r_1 \cdot r_2)$.
18. A particle moves along a circular path in such a way that its x – and y – coordinates at time t are $x = 2 \cos t, y = 2 \sin t$. Find the instantaneous velocity and speed of the particle at time t .
19. Define Level Surfaces of $f(x, y, z)$ and find the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.

20. For $f(x, y) = x^2y + 5y^3$, find the slope of the surface $z = f(x, y)$ in the x-direction in the y-direction at the point $(1, -2)$.
21. Let $f(x, y) = x^2e^y$. Find the maximum value of a directional derivative at $(-2, 0)$.
22. State the Second Partial test for finding the relative extrema of a function.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. **Each** question carries **4** marks.

23. Let a and b are any positive integers. Prove that the number of positive integers $\leq a$ and divisible by b is $[a \setminus b]$.
24. Prove that every integer $n \geq 2$ has a prime factor.
25. Prove that two integers a and b are relatively prime if and only if there are integers α and β such the $\alpha a + \beta b = 1$.
26. Prove that, if $r(t)$ is a vector-valued function and $r(t)$ is differentiable at t , then $r'(t) = x'(t)i + y'(t)j$. Also for vector-valued functions $r_1(t)$ and $r_2(t)$, Prove that $\frac{d}{dt}[r_1(t) + r_2(t)] = \frac{d}{dt}[r_1(t)] + \frac{d}{dt}[r_2(t)]$.
27. If $r(t)$ is a differentiable vector-valued function in 2-space or 3-space and $\|r(t)\|$ is constant for all t , then $r(t) \cdot r'(t) = 0$.
28. Give the formula for arc Length L from $t = a$ to $t = b$, for the graph of a smooth vector-valued function $r(t)$. Also Find the arc length of that portion of the circular helix, $x = \cos t, y = \sin t, z = t$ from $t = 0$ to $t = \pi$.
29. Assuming that polynomials in one variable and sine function are continuous, show that $f(x, y) = \sin(3x^2y^5)$ is continuous everywhere State the results used for the proof.
30. Prove that if a function $f(x, y)$ is differentiable at a point, then it is continuous at that point.
31. Suppose that $w = \sqrt{x^2 + y^2 + z^2}, x = \cos \theta, y = \sin \theta, z = \tan \theta$. Use the chain rule to find $\frac{dw}{d\theta}$ when $\theta = \frac{\pi}{4}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) State and prove the Division Algorithm for integers.

(b) Using Euclidean Algorithm express (4076, 1024) as a linear combination of 4076 and 1024.

33. (a) Find $T(t)$ and $N(t)$ for the circular helix $x = a \cos t$, $y = a \sin t$, $z = ct$ where $a > 0$.

(b) Find the curvature for the ellipse $r = 2 \cos t \mathbf{i} + 3 \sin t \mathbf{j} (0 \leq t \leq 2\pi)$.

34. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^4 \sin(xy^3)$. Given $f(x, y) = -\frac{xy}{x^2 + y^2}$, Find the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ limit along.

(a) the x-axis

(b) the y-axis

(c) the line $y = x$

(d) the line $y = -x$

(e) the parabola $y = x^2$

35. Explain Lagrange's Multiplier method for finding extreme values and find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point (1, 2, 2).

(2 × 15 = 30 Marks)