

Reg. No. :

Name :

Fourth Semester B.Sc. Degree Examination, May 2021

First Degree Programme under CBCSS

Mathematics

Core Course III

MM 1441 : ELEMENTARY NUMBER THEORY AND CALCULUS II

(2018 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They carry 1 mark each.

1. The linear congruence $ax \equiv b(\text{mod } m)$ has a unique solution if and only if _____.
2. Give a counter example to show that $(a,m)=(b,m)$ does not imply that $a \equiv b(\text{mod } m)$.
3. Determine whether 61, 194, 858, 376 is a square.
4. Compute $\phi(18)$.
5. State Fubini's theorem.
6. Evaluate $\int_0^1 xy^2 dy$.

7. Let T be the transformation from the uv -plane to the xy -plane defined by the equations $x = 14(u + v)$, $y = 12(u - v)$. Find $T(1, 3)$.
8. Find parametric equations for the surface generated by revolving the curve $y = 1/x$ about the x -axis.
9. State true or false. If F is a vector field, then so is $\nabla \times F$.
10. Find the vector field F whose potential function $\phi(x, y, z) = xy + yz + xz$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions among the questions **11 to 22**. They questions carry **2** marks each.

11. Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares.
12. Prove that a palindrome with an even number of digits is divisible by 11.
13. Find the remainder when $1! + 2! + \dots + 1000!$ is divided by 10.
14. Find the least residues x such that $x^2 \equiv 1 \pmod{8}$.
15. Evaluate the triple integral $\iiint_G 12xy^2z^3 dv$ over the rectangular box G defined by the inequalities $-1 \leq x \leq 2$, $0 \leq y \leq 3$, $0 \leq z \leq 2$.
16. Use a double integral to find the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0, 1] \times [0, 2]$.
17. Express the integral $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} dx dy$ as an equivalent integral with the order of integration reversed.

18. Find the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ of the transformation $x = uv, y = vw, z = 2w$.
19. Sketch the gradient field of $\phi(x, y) = x + y$.
20. Find $\text{div } F$ and $\text{curl } F$ for the vector field $F(x, y, z) = x^2i - 2j + yzk$.
21. Evaluate $\int_C (3x^2 + y^2)dx + 2xy dy$ along the circular arc C given by $x = \cos t$, $y = \sin t (0 \leq t \leq \pi/2)$.
22. Let $F(x, y) = (ye^{xy} - 1)i + xe^{xy}j$. Find a potential function for F .

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions among the questions **23** to **31**. These questions carry **4** marks each.

23. Find the positive integers n for which $\sum_{k=1}^n k!$ is a square.
24. Find the remainder when 3^{247} is divided by 17.
25. Solve the congruence $12x \equiv 48 \pmod{18}$.
26. Using the pollard $(p-1)$ method, find a nontrivial factor of $n = 2813$.
27. Let G be the wedge in the first octant that is cut from the cylindrical solid $y^2 + z^2 \leq 1$ by the planes $y = x$ and $x = 0$. Evaluate $\iiint_G z dV$.
28. Find an equation of the tangent plane to the parametric surface $x = uv, y = u, z = v^2$ at the point where $u = 2$ and $v = -1$.
29. Evaluate $\iint_R \sin \theta dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos \theta)$.

30. Find the work done by the force field $F(x, y) = (e^x - y^3)i + (\cos y + x^3)j$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction using Green's theorem.
31. Evaluate the surface integral $\iint_{\sigma} xz dS$, where σ is the part of the plane $x + y + z = 1$ that lies in the first octant.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions among the questions 32 to 35. These questions carry 15 marks each.

32. (a) State and prove Chinese Remainder Theorem.
- (b) Solve the linear system $x \equiv 1(\text{mod } 3)$, $x \equiv 2(\text{mod } 4)$, and $x \equiv 3(\text{mod } 5)$.
33. (a) Find the volume of the region G enclosed by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
- (b) Use spherical coordinates to evaluate $\int_{-3}^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} dz dx dy$.
34. (a) Evaluate $\iint_R \frac{x-y}{x+y} dA$, where R is the region enclosed by $x = y = 0$, $x - y = 1$, $x + y = 1$ and $x + y = 3$.
- (b) Use the Divergence theorem to find the outward flux of the vector field $F(x, y, z) = x^3i + y^3j + z^2k$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.
35. State Stoke's theorem and verify it for the vector field $F(x, y, z) = 2zi + 3xj + 5yk$, taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation, and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy -plane.

(2 × 15 = 30 Marks)