

SECTION IV

Answer any **two** questions. **Each** question carries **15** marks.

32. Find a root of the equation $2x^3 - 2x - 5 = 0$ using bisection method correct to two decimal places.

33. Find the reduced row echelon form of $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 4 & 3 & 4 & 0 \end{pmatrix}$.

34. Solve $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$.

35. Solve (a) $\frac{(x+1)dy}{dx} - y = e^{3x}(x+1)^2$ (b) $(y - x^3)dx + (x + y^3)dy = 0$.

(2 × 15 = 30 Marks)

(Pages : 4)

S – 3419

Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III-DIFFERENTIAL EQUATIONS, THEORY OF EQUATIONS AND THEORY OF MATRICES

(2014-2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION I

All the first ten questions are compulsory. Each question carries 1 mark.

1. Write the matrix notation for the system of equations $2x_1 - x_2 + 3x_3 + x_4 = 1$, $x_1 + 3x_2 - 2x_4 = 0$, $-4x_1 - x_2 + 2x_3 - 9x_4 = -3$.
2. Define the rank of a matrix.
3. What is meant by the fundamental theorem of algebra?
4. Give an example for a symmetric matrix.
5. Let A be a square matrix of order n having n distinct eigen values, then is it diagonalisable?
6. When do we say that a square matrix is a diagonal matrix?

7. What is the general solution of the linear equation $\frac{dy}{dx} + Py = Q$?
8. State the test for exactness for the first order differential equation $Mdx + Ndy = 0$, where M, N are functions of x and y .
9. Define orthogonal trajectories of a family of curves.
10. Evaluate $\frac{1}{D} e^{2x}$ where $D \equiv \frac{d}{dx}$.

(10 × 1 = 10 Marks)

SECTION II

Answer any **eight** questions. Each question carries **2** marks.

11. Show that $x^4 + qx^2 + rx - s = 0$ has two imaginary roots, where q, r and s are positive real numbers.
12. Determine whether AB is defined for the matrices $A = \begin{pmatrix} -4 & -2 & 0 \\ 0 & 5 & 3 \\ -3 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -3 & 4 \end{pmatrix}$.
13. What is the rank of $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{pmatrix}$?
14. When do say that a homogeneous system of linear equations has infinitely many solutions?
15. Find $|A|$ if A is an orthogonal matrix.
16. Find the characteristic polynomial of $\begin{pmatrix} -5 & 0 \\ 1 & 2 \end{pmatrix}$.
17. Let A be a matrix of order $n \times m$. Then what will be the dimension of the solution space of the system $AX = 0$.
18. Find the augmented matrix corresponding to the system of equations $-3x + 2y + 2z = 8$, $x + 4y - 6z = 1$, $2z - 2y = -2$

19. What is the order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = c^2 \left(\frac{d^2y}{dx^2}\right)^2$?
20. Verify that $y = \frac{c}{x}$ is a solution of the differential equation $xy' = -y$.
21. Write the standard form for a homogeneous linear ODE of second order.
22. Find a particular integral of $(D^2 + 5D + 6)y = e^x$.

(8 × 2 = 16 Marks)

SECTION III

Answer any **six** questions. Each question carries **4** marks.

23. What is meant by the reduced row echelon matrix?
24. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$.
25. Check whether the matrix $\begin{pmatrix} 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 1 & 0 & 0 \\ 0 & \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{pmatrix}$ is an orthogonal matrix.
26. Show that $y = -\sin x + ax^2 + bx + c$ is a solution of $y''' = \cos x$.
27. Show that $\frac{dy}{dx} = e^{3x-2y} - x^2e^{-2y}$ is separable and then solve it.
28. Solve the initial value problem $y'' + y = 0$, $y(0) = 3.0$, $y'(0) = -0.5$.
29. Solve $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$.
30. Solve $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$.
31. Solve $x^4 - 4x^2 + 8x + 35 = 0$, given $2 + i\sqrt{3}$ is a root.

(6 × 4 = 24 Marks)