

Reg. No. :

Name :

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Core Course X

MM 1642 : LINEAR ALGEBRA

(2015-2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. Each carries 1 mark.

1. Write the equation of the line passing through $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and parallel to the vector $B - A$.
2. Evaluate the polar angle of the vector $X = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.
3. Find the projection of $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ to the line along $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$.
4. Find the transformation P which assign to each vector $X = \begin{pmatrix} x \\ y \end{pmatrix}$ the reflection of X on the line along the vector $U = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
5. True/False: If A and B are two linear transformations, then $\det(AB) \neq (\det B)(\det A)$ for some A and B .

6. True/False: A real number 't' is a root of the characteristic equation of a linear transformation A if 't' is an eigen value of A.
7. What is the 100^{th} power of the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 10 \end{pmatrix}$
8. Check whether $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, are linearly independent.
9. Show that $(tA) \times A = t(A \times B)$, for two vectors A and B in 3-Space.
10. Write an example of a hyperplane in $\mathbb{R}^4$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from this Section. Each carries **2** marks.

11. Show that if S, T are linear transformations, then ST and TS are linear transformations.
12. Find the angle between $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.
13. Show that $J_k J_m = J_{k+m}$, where J_k is the transformation with matrix $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$.
14. Let A be a linear transformation with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, if the only vector X with $A(X) = 0$ is the vector $X = 0$, then show that $ad - bc \neq 0$.
15. If P is the projection to the x_1, x_2 -plane and Q be the projection to the $x_2 x_3$ -plane. Then show that $PQ = QP$.
16. Show that $(e_{31}^s)^{-1} = e_{31}^{-s}$.
17. Prove that if S and T are linear transformations of $\mathbb{R}^3$ such that $ST = I$, then $TS = I$.

18. Show that if the three row vectors A, B, C of a matrix M are linearly dependent then $\det(M) = 0$.

19. Show that the endpoints of the five basis vectors in $\mathbb{R}^5$ form the vertices of a regular 4 simplex.

20. Find a real number t such that $\begin{pmatrix} 1 \\ 2 \\ 1 \\ -1 \end{pmatrix}$ is orthogonal to $\begin{pmatrix} t \\ -t \\ 1-t \\ 2t-t \end{pmatrix}$.

21. Show that the basis vectors $\{E_1, E_2, E_3, E_4\}$ forms an orthonormal set.

22. Find the reflection through the line along $U = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$ in $\mathbb{R}^4$

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from this Section. Each carries **4** marks.

23. Find all solutions to the system of equations

$$\begin{aligned} 5x + y &= 0 \\ -10x - 2y &= 0 \end{aligned}$$

24. Let A be a linear transformation and let L be a straight line. Then prove that the image of L under A is either a straight line or a single point.

25. Find the inverse of $R_{3\pi/4}$.

26. Prove that 2 vectors A and B are linearly dependent if and only if $A \times B = 0$.

27. Prove that, if A, B, C are three linearly independent vectors in $\mathbb{R}^3$, then every vector can be uniquely expressed as a linear combination of A, B and C .

28. If T is an isometry on $\mathbb{R}^3$, then prove that $\det(T) = 1$ or $\det(T) = -1$

29. Classify the quadratic surface $x^2 + y^2 - z^2 = 0$.

30. Show that the three vectors $\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix}$, $\begin{bmatrix} -y \\ x \\ -v \\ u \end{bmatrix}$, $\begin{bmatrix} u \\ -v \\ -x \\ y \end{bmatrix}$ are mutually orthogonal and all have the same length.

31. Find all vectors in $\mathbb{R}^4$ which are orthogonal in $\mathbb{R}^4$ to the vector $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$
(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from this Section. Each carries **15** marks.

32. (a) Let A be a linear transformation with symmetric matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ and; let t_1, t_2 be distinct eigenvalues of A . Choose non-zero eigenvectors X_1, X_2 corresponding to t_1 and t_2 respectively. Then show that X_1 and X_2 are orthogonal.
- (b) For the above linear transformation A , show that $A(X) \cdot Y = X \cdot A(Y)$ for any two vectors X and Y .
33. (a) Let T be a linear transformation of $\mathbb{R}^3$ and denote by A, B, C the row vectors of the matrix of T . Then prove that T has an inverse if and only if $A \cdot (B \times C) \neq 0$.
- (b) If $m(T) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ find $m(T^{-1})$
34. State and prove spectral theorem in $\mathbb{R}^3$.
35. (a) Prove that a transformation T of 4-space is a linear transformation if and only if $T(X+U) = T(X) + T(U)$ and $T(rX) = rT(X)$ for any vectors X and U and scalars r .
- (b) Define a 4×4 determinant in terms of 3×3 determinants in $\mathbb{R}^4$.

(2 × 15 = 30 Marks)